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Ansgar Belke
Matthias Göcke
Laura Werner

Hysteresis Effects in Economics – Different Methods for Describing Economic Path-dependence

UNIVERSITÄT DUISBURG ESSEN

#468

Imprint

Ruhr Economic Papers

Published by

Ruhr-Universität Bochum (RUB), Department of Economics
Universitätsstr. 150, 44801 Bochum, Germany

Technische Universität Dortmund, Department of Economic and Social Sciences
Vogelpothsweg 87, 44227 Dortmund, Germany

Universität Duisburg-Essen, Department of Economics
Universitätsstr. 12, 45117 Essen, Germany

Rheinisch-Westfälisches Institut für Wirtschaftsforschung (RWI)
Hohenzollernstr. 1-3, 45128 Essen, Germany

Editors

Editorial Office

Sabine Weiler
RWI, Phone: +49 (0) 201/81 49-213, e-mail: sabine.weiler@rwi-essen.de

Ruhr Economic Papers #468

Responsible Editor: Volker Clausen

ISSN 1864-4872 (online) – ISBN 978-3-86788-528-7

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Bibliografische Informationen der Deutschen Nationalbibliothek

Die Deutsche Bibliothek verzeichnet diese Publikation in der deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über: *http://dnb.d-nb.de* abrufbar.

http://dx.doi.org/10.4419/86788528
ISSN 1864-4872 (online)
ISBN 978-3-86788-528-7

Ansgar Belke, Matthias Göcke, and Laura Werner[1]

Hysteresis Effects in Economics – Different Methods for Describing Economic Path-dependence

Abstract

Relations between economic variables are often characterized by a situation where initial conditions and the past realizations of economic variables matter. I.e. past (transient) exogenous disturbances and past states of the economic system do have an influence on the current economic relations. Typical examples are the dynamics of (un)employment in business cycles and the dynamics of the nexus of exchange rate and exports. Since the standard characteristics of hysteresis apply – i.e. permanent effects of a temporary stimulus, resulting in path-dependent multiple equilibria – these economic phenomena are correctly titled as "hysteresis". Empirical research in economics is using different methods in order to capture pathdependent effects. First econometric approaches tried to describe these effects by simple timeseries processes with unit- (or zero)-root dynamics. However, since unit-root-dynamics are not related to genuine multiple equilibria but on the order of integration of the time series, these first attempts were expanded by more sophisticated time-series models integrating structural breaks, threshold-cointegration or non-linear autoregressive distributed lag-models. Another branch of empirical studies tries to keep closer to the original concept of the macroloop, trying to apply an explicit Mayergoyz/Preisach aggregation procedure for heterogeneous firms – if microeconomic information is available based on panel-data – or by using simple algorithms analogous to mechanical-play in order to apply simple OLS-regression methods on a filtered/transformed input-output relation. In this paper, we give an overview of the implementation of hysteresis in economics, with an emphasis on two aspects: (1) the differentiation between micro- and macroeconomic hysteresis including an outline of an adequate aggregation procedure, and (2) different methods applied in econometrics in order to capture economic path-dependency empirically.

JEL Classification: C51, C62

Keywords: Play-hysteresis; modelling techniques; switching/spline regression; path-dependence

January 2014

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[1] *Ansgar Belke, IZA Bonn and UDE; Matthias Göcke and Laura Werner, Justus Liebig University of Giessen. – All correspondence to: Ansgar Belke, University of Duisburg-Essen, Chair for Macroeconomics, Department of Economics, 45117 Essen, Germany, e-mail: ansgar.belke@uni-due.de*

1. Introduction

Relations between economic variables are often characterized by a situation where initial conditions and the past realizations of economic variables matter. I.e. past (transient) exogenous disturbances and past states of the economic system do have an influence on the current economic relations. Typical examples are the dynamics of (un)employment in business cycles, i.e. the dynamics of the so called "natural" (equilibrium) rate of unemployment, and the dynamics of the nexus of exchange rate and exports. Since the standard characteristics of hysteresis apply – i.e. permanent effects of a temporary stimulus, resulting in path-dependent multiple equilibria – these economic phenomena are correctly titled as "hysteresis" (*Cross, Allan*, 1988). In labor economics applying the concept of hysteresis was especially disseminated by *Phelps* (1972), *Sachs* (1986), *Blanchard, Summers* (1986) and *Lindbeck, Snower* (1986). To foreign trade theory hysteresis was introduced by *Kemp, Wan* (1974), *Baldwin* (1989), *Baldwin, Krugman* (1989) and *Dixit* (1989).

Analogous to magnetism, the pattern of hysteresis depends on the scope: Based on sunk-adjustment costs (e.g. entry costs of starting export activity on foreign markets) *micro*economic behavior (e.g. of *single firms* on export markets) shows a discontinuous switching-pattern (being active on a foreign market or not) as described by a non-ideal relay, analogous to the magnetism of a single iron crystal. Correspondingly, *macro*economic dynamics of *aggregate* economic variables (e.g. the exports volume of a *whole country,* based on an aggregation over firms with heterogeneous cost structures) show a pattern similar to the well known hysteresis-loop of an entire piece of iron. The aggregate macroeconomic loop is characterized by a smooth/continuous transition between different "branches" of the loop, occurring with changes in the direction of the (e.g. exchange rate) movement.

Empirical Research in economics is using different methods in order to capture path-dependent effects. First econometric approaches tried to describe these effects by time-series processes with unit- or zero-root dynamics. However, since unit-root-dynamics are not related to genuine multiple equilibria but on the order of integration of the time series, these first attempts were expanded by more sophisticated time-series models integrating structural breaks, threshold-cointegration or non-linear autoregressive distributed lag-models. Another branch of empirical studies tries to keep closer to the original concept of the macro-loop, trying to apply an explicit *Mayergoyz* (1986, 2003)/*Preisach* (1935) aggregation procedure for heterogeneous firms – if microeconomic information is available based on panel-data – or by using simple algorithms analogous to mechanical-play in order to apply simple OLS-regression methods on a filtered/transformed input-output relation.

In the following chapter neither a detailed description of the various techniques nor a comprising presentation of all manifold applications can be presented (see the references for this purpose). Rather, we want to illustrate the character of hysteresis based on standard examples including their economic intuition in order to give an overview of the distinct concepts and way to implement hysteresis in economics. This overview emphasizes two aspects: First (in Section 2) the differentiation between micro- and macroeconomic hysteresis including an outline of an adequate aggregation procedure, and second (in Section 3) different methods applied in econometrics in order to capture economic path-dependency empirically.

2. Theoretical background for hysteresis in economics

2.1 Microeconomic hysteresis based on sunk costs

A change of the relevant forcing variables typically leads to a change in the economic behavior of the observed unit(s). However, in the case of hysteresis a removal of a merely temporary change back to the initial value of the forcing variables does not induce a complete change back to the initial behavior. A typical economic mechanism for this path-dependent pattern is founded on *sunk adjustment costs* (see *Baldwin*, 1989, 1990).

The typical microeconomic example is as follows: A firm which is previously not selling on a market has to bear market entry costs, e.g. for setting up distribution networks or for advertising, which are firm specific and cannot be regained after market entry. Thus, ex-post these market entry ‘investments’ are sunk costs. A market entry is only profitable, if the sunk entry costs are covered by revenues, thus, the market price must exceed the unit costs for triggering market entry. If a temporary high market price has led to a market entry of a firm, a subsequent removal of this price increase back to the initial level will – as long as the variable costs are covered – not induce a market exit. Summarizing, the same price level may result in different states of the firm’s activity, dependent on the history of the price level.

Adjustment costs may occur on the supply side of the market (as outlined above in the simple example) or on the demand side of a market (see *Froot, Klemperer,* 1989, p. 638, for a systematization in supply and demand side of factors generating hysteresis): Another important supply side reason is based on learning-by-doing, since former production and gaining experience leads to higher productivity and, thus, later on to lower costs. On the demand side, experience effects of consumers may lead to demand-carry over effects; i.e. a temporary price decrease, inducing additional sales and a higher willingness to pay may result in a permanently higher demand, if consumers accumulate positive experience with the firm's product. The defining aspect is that, though all these factors play only a transient role, the result is a difference between the ex-ante decision (before e.g. the sunk costs are paid) and the ex-post situation (after these 'investments' are carried out). As the relevant marginal costs or revenues were changed, the same exogenous environment path-dependently results in different reactions. Thus, a transient disturbance can have permanent effects on economic equilibria – which is the constituting characteristic of economic hysteresis.

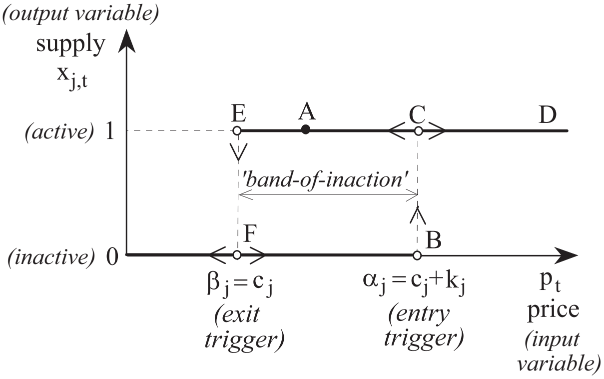
'Non-ideal relay'-hysteresis in economics

In order to illustrate an economic situation which is characterized by the most elementary form of hysteresis – the so called 'non-ideal relay' – we model the simple introductory microeconomic example (*Göcke*, 2002): A firm j decides in period t whether or not to supply one unit of a product ($x_{j,t}=1$ or $x_{j,t}=0$). For producing this unit "variable" costs c_j have to be paid, and additionally, if the firm has not produced and sold on the market in the preceding period (t–1), stepped fixed starting costs k_j. This sunk 'investment' k_j is completely firm specific and decays immediately when the firm stops producing. Hence, the costs of an active firm is path-dependent: If inactive in the preceding period (production/sales $x_{j,t-1}=0$) it has to pay both components, but if it was active before (supply $x_{j,t-1}=1$) only the variable part is relevant. The firm's (unit) cost function is:

$$(1) \qquad K_{j,t} = \begin{cases} 0 & \text{if } x_{j,t}=0 \\ c_j + k_j & \text{if } x_{j,t}=1 \wedge x_{j,t-1}=0 \\ c_j & \text{if } x_{j,t}=x_{j,t-1}=1 \end{cases} \qquad \text{with } c_j, k_j \geq 0$$

If the (price-taking) firm is (for the time being) assumed to be myopic, i.e. only looking at the periods costs and revenues and not considering the future advantage of being active in the current period, it compares the current price p_t with the current costs. The resulting supply behavior of the firm is illustrated in Fig. 1: If the (forcing variable) price p_t starts to rise continuously from a zero level, the (previously inactive) firm starts producing/selling, if the price exceeds the sum of both costs $\alpha_j=(c_j+k_j)$ as an entry *trigger* in point B. Market entry leads to a jump from the (x_j=0)-inactivity-line to the (x_j=1)-activity-line (point C). If the firm in later periods faces a price decrease (e.g. in point A), it will continue to produce as long as the variable costs (as an exit trigger $\beta_j=c_j$) are covered – since ex-post the sunk entry cost k_j are not relevant any more. The variable costs are the exit trigger. The lowest-price limit of the firm depends on past market/production activity. Two different path-dependent equilibria are possible between points F and B (or E and C, respectively). A switch from one equilibrium-branch to the other takes place when the triggers are passed – otherwise the activity status remains the same. Therefore, the area FB (or CE) can be described as a *'band of inaction'* (*Baldwin*, 1989, pp. 7 f., and *Baldwin, Lyons*, 1989, p. 11.). This elementary form of hysteresis is called *'non-ideal relay'* by *Krasnosel'skii, Pokrovskii* (1989, p. 263 and p. 271), since the switch between the two possible equilibria is triggered by *two different* values (*Brokate, Sprekels*, 1996, pp. 23 f.).[1] Since for a non-ideal relay only a transition between two branches is possible, a major characteristic of hysteresis – the *'multibranch non-linearity'* – exists in a very simple way (*Mayergoyz*, 1986, p. 603).

Fig. 1: 'Non-ideal relay' of a firm with sunk entry costs



Passing of microeconomic triggers usually results from "large shocks". Thus, papers implicitly relying on "band-of-inaction"-type hysteresis, point out the difference between large shocks triggering permanent effects and small ones that do not. (See e.g. *Baldwin, Krugman,* 1989; *Baldwin, Lyons*, 1989, and *Evans, Honkapohja*, 1993).

The ex-ante/ex-post difference of the relevant costs and revenues leading to a band-of-inaction applies in an analogous way to different markets. The important example of labor demand is briefly outlined in the following. Potential mechanisms inducing labor market hysteresis are:

- Hiring and firing costs (as sunk employment adjustment cost): the employer has to pay for hiring and training new employees; and firing costs occur e.g. for severance pay or due to

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[1] On a micro level, i.e. on the level of a single iron-crystal, magnetic hysteresis shows exactly this pattern (*Kneller*, 1962, pp. 401 f.).

the loss of firm specific human capital of the fired employees. Both hiring and firing costs could not be regained after the employment adjustment is executed.

- Due to learning effects, the productivity of workers is rising in the course of a longer employment. In contrast, unemployment leads to a depreciation of the quality of labor due to a lack of training-on-the-job. These mechanisms are increasing the willingness-to-pay for active (already employed) workers relative to unemployed workers. Thus, the time-path of employment is determining individual wages and employment opportunities.

Thus situations triggering hiring are different from the level that triggers firing, i.e. a hysteresis band of inaction arises: the wage rate that induces hiring is lower than the wage rate that causes firing. Consequently, transient labor market disturbances can have permanent effects on the so called "natural" equilibrium rate of unemployment.

"Mechanical play" in microeconomics

Another very simple type of hysteresis – which is in economic modelling not as common as the non-ideal relay pattern – is a dynamics similar to the phenomenon of mechanical play.[2] In microeconomics 'play-hysteresis' results from *variable* sunk adjustment costs. The firm j is now able to produce and sell the variable amount $x_{j,t} \geq 0$, based on the cost function:

$$(2) \qquad K_{j,t} = c_j \cdot x_{j,t}^2 + k_j \cdot D \cdot (x_{j,t} - x_{j,t-1}) \qquad \text{with: } D = \begin{cases} 0 & \text{if } x_{j,t} \leq x_{j,t-1} \\ 1 & \text{if } x_{j,t} > x_{j,t-1} \end{cases}$$

In addition to the standard average variable costs $(c_j \cdot x_{j,t})$, the firm has to pay *variable adjustment costs* for an increase $(x_{j,t} - x_{j,t-1}) > 0$ of *production*. Adjustment costs are linear in the size of the increase, i.e. k_j per unit of the expansion. To keep the model simple, again a "myopic" firm – only considering current payments – is assumed. Previous production $(x_{j,t-1})$ determines the "capacity", which is costly to be increased – and which has to be written of completely if not employed for one period. If the price level p_t is exogenously given, i.e. if the firm is a price taker and only adjusts its quantity according to profit maximization (based on the rule "marginal revenue = price = marginal costs"):

$$(3) \qquad p_t = 2 \cdot c_j \cdot x_{j,t} + k_j \cdot D \qquad \Rightarrow \qquad \begin{cases} p_t = 2 \cdot c_j \cdot x_{j,t} & \text{if } x_{j,t} \leq x_{j,t-1} \\ p_t = 2 \cdot c_j \cdot x_{j,t} + k_j & \text{if } x_{j,t} > x_{j,t-1} \end{cases}$$

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[2] See *Göcke* (2002), for a more comprehensive presentation of the following example. See *Krasnosel'skii, Pokrovskii* (1989, pp. 6 ff.) and *Brokate, Sprekels* (1996, pp. 24 f. and pp. 42 ff.) for a general treatment of the play-operator. For an example of play-hysteresis in economics see *Delgado* (1991, Fig. 2, p. 472) where price-stickiness as a result of menu-costs is analysed.

Marginal costs are different in situations with expanding production compared to situations with constant or decreasing production: the conditional optimum for increasing production $x_{j,t}^{up*}$ (including capacity adjustment costs k_j) and $x_{j,t}^{do*}$ as the optimum conditional on constant or decreasing production:

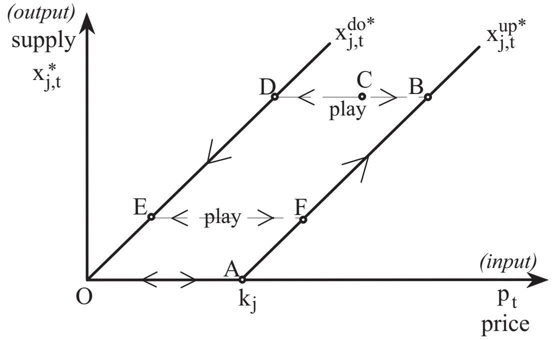
$$(4) \qquad x_{j,t}^{up*} = \frac{p_t - k_j}{2 \cdot c_j} \qquad \text{and} \qquad x_{j,t}^{do*} = \frac{p_t}{2 \cdot c_j} \qquad\qquad \text{with: } x_{j,t}^{do*} > x_{j,t}^{up*} \Leftrightarrow k_j > 0$$

The actual optimum $x_{j,t}^{*}$ results from the comparison of the two conditional optima with the past production $x_{j,t-1}$:

$$(5) \qquad x_{j,t}^{*} = \begin{cases} x_{j,t}^{up*} = \dfrac{p_t - k_j}{2 \cdot c_j} & \text{if } x_{j,t}^{up*} > x_{j,t-1} \\ x_{j,t-1} & \text{if } x_{j,t}^{up*} \le x_{j,t-1} \le x_{j,t}^{do*} \\ x_{j,t}^{do*} = \dfrac{p_t}{2 \cdot c_j} & \text{if } x_{j,t}^{do*} < x_{j,t-1} \end{cases}$$

Fig. 2 illustrates the resulting play-hysteresis loop of the optimal (output) $x_{j,t}^{*}$ for changes in the current price p_t (as input variable). Starting (in the origin O) with a zero price and a zero capacity ($x_{j,t-1}=0$), a price increase will induce no production (increase) until in point A a threshold is exceeded (where enlargement costs k_j are covered). For larger price increases the production reacts according to line AFB. If price rises up to point B, the production increases as well. However, for a subsequent price decrease, the production shows no reaction along the line BCD, until a threshold is passed in point D. As in the case of a non-ideal relay, no reaction of the output variable occurs for a particular input area. However, in the case of play the position of the inaction-band is itself path-dependent (and not invariant as for a non-ideal relay). Initially, its position is defined by the line OA and later by BD (or by EF).

Fig. 2: A microeconomic reaction similar to ‘mechanical play’



Play is represented by the horizontal distance between the right line AFB (describing a production increase) and the left OED line (representing decreasing production generated by a price decline). As a consequence of *variable* sunk adjustment (capacity enlargement) costs, there is a difference between the two conditional optima $x_{j,t}^{up*}$ and $x_{j,t}^{do*}$, resulting in a constant supply of the firm if the direction of the price/quantity movement is changed – until the thresholds, determined by the past production $x_{j,t-1}$, are passed.

Both microeconomic hysteresis loops, non-ideal relay and play, show some common features: multibranch non-linearity is characterized by two main branches and the change between these branches requires passing of thresholds, while the passing of the thresholds results in permanent effects. However, for play-loops the change is not discontinuously abrupt but smoothly. However, the non-ideal model case can be interpreted as a special case of the play model where only a discrete (0,1) supply reaction is feasible.

Uncertainty and option value effects

A forward-looking firm considers future effects of a present sunk cost 'investment'. If the exogenous variable (price) is stochastic, a real option approach applies (*Pindyck*, 1988 and 1991; *Dixit*, 1989; *Bentolila, Bertola*, 1990; *Belke, Göcke*, 2001). An inactive firm deciding on a present entry or to stay passive, will include the option to enter later as a potential alternative. A price which is presently covering costs, may in a stochastic situation, decrease in the future. By staying passive the firm can avoid future losses if this situation will realize. Moreover, an instantaneous entry kills the option to enter later and to "wait-and-see" if the future price movement will turn out to be (un)favorable. Thus, in a stochastic situation, the sunk costs and, additionally, an option value of waiting have to be covered in order to trigger an entry. Thus, uncertainty implies an upward shift of the entry trigger price.

Consider a firm with an infinite planning horizon. Again the previously inactive price taker firm j has to decide in period t whether or not to enter, paying sunk costs k_j, and to supply one unit, produced with costs c_j. Assume a non-recurring single stochastic change in the price level in the next period (t+1), which can be either positive (+u) or negative (–u) (with $u \geq 0$), with the same probability of ½ : $p_{t+1} = p_t \pm u$ and $E_t(p_{t+1}) = p_t$: After this shock, the price is assumed to remain constant for the rest of the infinite future. Under uncertainty, the three alternatives which have to be compared are (1) to enter now (in t), (2) to enter in the next period t+1, or (3) to enter never. A *present* entry in period t leads to an expected present value $EPV_{j,t}^{enter}$, if discounting is based on the interest rate $i > 0$ (and using the formula of the present value of an annuity due):

$$(6)\qquad EPV_{j,t}^{enter}=\frac{(1+i)\cdot(p_t-c_j)}{i}-k_j \qquad\qquad \Rightarrow\ EPV_{j,t}^{enter}>0\ \text{ if }\ p_t>\alpha_j^c=c_j+\frac{i}{1+i}\cdot k_j$$

Naively looking only at a positive present value of a present entry (neglecting the option to wait), would result in a certainty-equivalent entry trigger price $[\alpha_j^c=c_j+\frac{i}{1+i}\cdot k_j]$. However, additionally considering the option to wait, the firm differentiates between two potential u-realizations. If the firm waits in t, the firm can use its option to enter in t+1 conditional on a (+u)-realization (with probability ½). In case of (–u)-realization the firm will remain passive (and avoids losses). Consequently, the expected present value of the wait-and-see strategy $EPV_{j,t}^{wait}$ is, based on discounted values of entry costs and annuity (since now the conditional entry is one period later):

$$(7)\qquad EPV_{j,t}^{wait}=\frac{1}{2}\cdot\frac{1}{1+i}\cdot\left(\frac{(1+i)\cdot(p_t+u-c_j)}{i}-k_j\right)$$

With this present value of waiting in mind the firm will enter the market in period t, only if a current entry is more valuable than waiting:

$$(8)\qquad EPV_{j,t}^{enter}>EPV_{j,t}^{wait} \qquad\qquad \Leftrightarrow\ p_t>\alpha_j^u=c_j+\frac{i}{1+i}\cdot k_j+\frac{u}{1+2\cdot i}=\alpha_j^c+\frac{u}{1+2\cdot i}$$

An increase in uncertainty enlarges the option value to enter later, since with a larger u the potential payoff conditional on a positive price change increases, leaving the downside payoff unchanged, since the firm will not enter in t+1 if the price falls. As a result, the entry trigger price including option value effects is α_j^u augmented by the term $[u/(1+2\cdot i)]$. Thus, uncertainty leads to a widening of the band of inaction, aggravating the hysteresis property of the firm's behavior. However, the qualitative property of micro hysteresis as a non-ideal relay has not changed.

2.2 Aggregation and macroeconomic ("strong") hysteresis

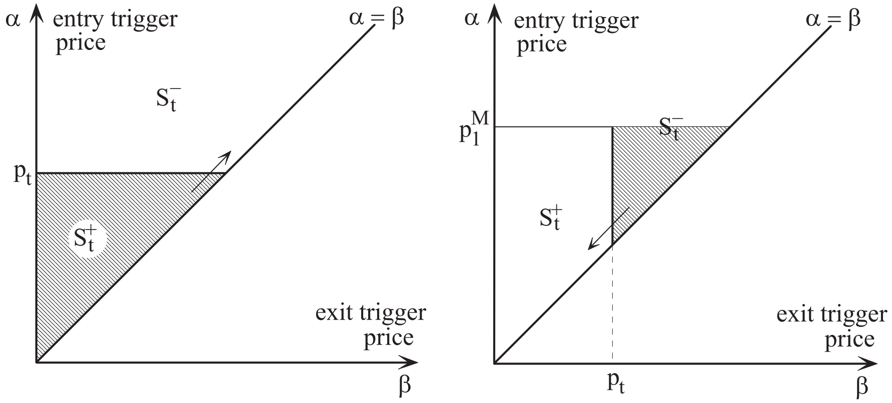
In the following the application of the *Mayergoyz (1986)/Preisach (1935)*-procedure is outlined. Being close to the original concept of hysteresis in magnetism, this method is based on the explicit aggregation of non-ideal relay agents ($j=1, ..., n$; $n \gg 0$) with heterogeneity in their cost structure, resulting in heterogeneous entry/exit triggers α_j and β_j (see *Amable et al.*, 1991, 1992, 1994; *Cross*, 1993, 1994; *Göcke*, 1994a, and *Piscitelli et al.*, 2000, and *Mota, Vasconcelos*, 2012, for applications of the *Preisach* model in foreign trade and in labor market economics).

Every potentially active firm j is characterized by a α_j/β_j-set of entry/exit triggers. In an α_j/β_j-diagram (since $\alpha_j \geq \beta_j$) all firms are represented by points in a triangle area above the 45°-line (see Fig. 3). Generally, the aggregation procedure can be performed without restrictions concerning the distribution of the firms over the triangle area. Points on the 45°-line represent non-hysteretic firms ($k_j = 0 \Rightarrow \alpha_j = \beta_j = c_j$). Firms with a position above the 45°-($\alpha = \beta$)-line are characterized by a non-ideal relay supply– the distance from the ($\alpha = \beta$)-line determined by k_j.

To avoid a long description of the past development, a simple situation with an initial price $p = 0$ is assumed, implying no firm is initially active. Now, a rising price leads to an entry of the firms with the lowest entry trigger $\alpha_j = (c_j + k_j)$. Aggregate supply increases, as traced in Fig. 3 (a), with a growing space of the hatched triangle S_t^+ representing the (active) firms which entered (and S_t^- representing the inactive firms). For a rising price the triangle S_t^+ grows via an upward shift of the horizontal borderline. The corresponding aggregate/macro supply reaction is delineated in Fig. 4 by the path OA.

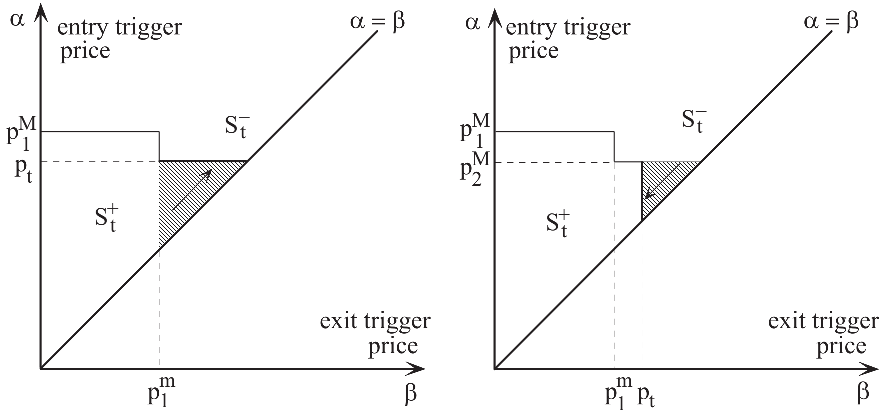
Fig. 3 (b) outlines the effects of a subsequent price decrease (after a local price maximum p_1^M was reached). Firms leave the market, if their exit triggers $ß_j$ are underbid and the triangle S_t^+ representing active firms now loses space as illustrated by a left vertical shift of the S_t^--S_t^+-borderline. In Fig. 4 the corresponding path is BC.

Fig. 3: *Preisach* aggregation procedure



(a) increasing price

(b) descending price

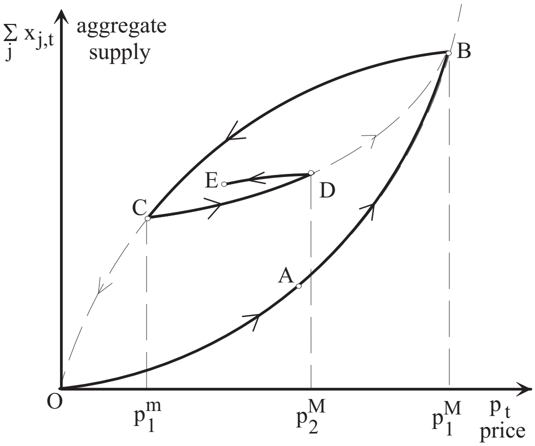


(c) later again increasing price

(d) later again decreasing price

If, after reaching the local minimum p_1^m, there is again a rise in the price level, as depicted in Fig. 3 (c) only the right-horizontal part of the S_t^+-borderline is shifted upwards (as long as the old maximum p_1^M is not passed), resulting in a macro reaction as path CD in Fig. 4. A subsequent price decrease is illustrated in Fig. 3 (d): The lower vertical part of the borderline is shifted to the left (corresponding to DE in Fig. 4). Several cycles of rising and decreasing prices will result in a "staircase"-borderline of the S_t^+-area of active firms, where the coordinates of the staircase are determined by past extrema of the price-movement. If later on the "old" local minima are underbid by even lower prices, or if local maxima are subsequently surpassed by higher prices, the corresponding staircase-corner (and the memory of the "old" extrema) is erased from the macro system (*Mayergoyz*, 1986, p. 605). If, however, a new local maximum p_2^M is lower than the “old” maximum p_1^M (as in Fig. 3 (c)), the memory of p_1^M remains.

Fig. 4: Aggregate macroeconomic hysteresis loop



Summarizing, the aggregate system shows a memory of non-erased (‘non-dominated’) past input extrema – graphically represented by the “staircases” in the borderline of the area S_t^+ of active firms. Aggregation results in a reinforcement in the type of hysteresis: For the aggregate loop a branch-to-branch transition occurs with *every local extremum* of the path of the input variable, and this transition is *continuously*, while at the microeconomic firm level the passing of triggers is necessary, causing a discontinuance switch/jump in order to induce permanent remanence effects. Therefore, this type of macro-hysteresis is called “*strong”* hysteresis (*Amable et al.* 1991, 1994; see *Brokate, Sprekels*, 1996, pp. 22 ff., for typical characteristics of the macro hysteresis loop). The *distribution* of the heterogeneous firms in the α,β-space determines the *curvature* of the branches of the macro loop. The more clustered the firms are in a specific area (i.e. the less heterogeneous the firms are), the more “curved” are the branches. In the limit case of homogeneous firms, these similar firms are all represented by a single point in the α/β-diagramm – and the macro-loop will degenerate to a non-ideal relay.

3. Approaches to address economic hysteresis empirically

The different types of hysteresis require specific modeling in empirical research. Thus, a variety of methods is used to describe permanent effects of temporary shocks empirically. In the next sections some econometric approaches that attempt to capture the hysteretic dynamics adequately are briefly outlined. A common method to describe persistence effects is by stochastic unit root difference equations (e.g. via cointegration models) which is adequate, if permanent effects are actually based on the degeneration of the adjustment dynamics. For non-ideal relay dynamics the switches between the branches may be addressed by (0,1)-dummy variables as additional variables in a regression equation. However, identifying the trigger values for the branch-to-branch transition remains a problem, since it has to be done as an endogenous part of the estimation (*Belke, Göcke*, 1997). Methods to integrate "strong"/aggregate hysteresis-loops in empirical models are e.g. presented by *Belke, Göcke* (1999a) and *Piscitelli et al.* (2000). These approaches attempt – based on past extrema – to calculate a transformed "hysteresis variable" to capture the dynamic pattern.

3.1 Time-series models

3.1.1 Linear difference equations with unit roots

As a simple way to describe persistent effects economists often try to describe hysteretic dynamics with models containing linear difference (differential) equations exhibiting unit (zero) roots (see *O'Shaughnessy*, 2000, as an example). Since zero root dynamics in continuous time can be considered analogously, here only unit root dynamics in discrete time are outlined explicitly. (For a general discussion of unit/zero roots and hysteresis see *Amable et al.*, 1992, 1993, and 1994; and *Hule*, 1996, pp. 43 ff.). However, unit root dynamics show some differences compared with "genuine" hysteresis. According to *Amable et al*. (1992, p. 8) unit root dynamics should be named 'persistence', but in economics this term is already applied for processes "close to" unit root dynamics, with a slow adjustment towards a unique equilibrium. Real hysteretic dynamics are rather captured by non-linear difference/differential equations (*Krasnosel'skii, Pokrovskii*, 1989, pp. 90 ff.; *Brokate, Sprekels*, 1996, pp. 122 ff.), and not by linear difference equations with unit roots as an oversimplification.

A single equation model with a unit root

Unit root dynamics are now illustrated by an inhomogeneous first order difference equation with constant coefficients, a state variable Y_t and only one exogenous variable R_t (*Franz*, 1990), using the lag-operator L:

$$(9) \qquad Y_t = b \cdot Y_{t-1} + R_t \quad \Rightarrow \quad Y_t - b \cdot L\, Y_t = R_t \quad \Rightarrow \quad (1 - b \cdot L) \cdot Y_t = R_t$$

The characteristic root of $[1 - b \cdot z = 0]$ is $z = (1/b)$. All roots have to be outside the complex unit circle, in order to have a stable steady state equilibrium (*Lütkepohl*, 1991, pp. 11 f.). Thus, for $0 < |b| < 1$ the steady state (with: $Y^* = Y_t = Y_{t-1}$ and $R^* = R_t = R_{t-1}$) results:

(10) $$Y^* = \frac{1}{1-b} \cdot R^*$$

For $0<b<1$ a stable equilibrium and for $b>1$ an explosive non-equilibrium path results. However, for $b=1$, there is a unit root ($z=1$), resulting in:

(11) $$Y_t = Y_0 + \sum_{\tau=1}^{t} R_\tau$$

Thus, for a non-constant exogenous variable ($R^* \neq 0$) a constant steady state Y* results in the case of $0<b<1$. In a unit root situation ($b=1$) the state variable only remains constant if no exogenous impulse occurs (i.e. $R=0$). In econometrics, a time series with a unit-root is called to be "*integrated of order one*". As in the case of hysteresis, the unit-root system is path-dependent, as the current state under a unit root depends on the initial conditions and on the past realizations of the exogenous variable. However, there are some differences: In the case of "genuine" hysteresis there is a path dependence of *locally stable* equilibria (under "slow control", after adjustment has taken place), while unit-root dynamics the permanent effects are based on an *indifference between equilibria*, i.e. a *degeneration of the adjustment dynamics*. A branch-to-branch transition in genuine hysteresis results from a *local instability* resulting in path-dependent *structural breaks* (*Baldwin*, 1990a, p. 245 ff.). For unit root dynamics a temporary shock results as well in a different equilibrium, but this is based on *global indifference* between different equilibrium states (*Amable et al.*, 1991, pp. 12 ff.). Due to this difference in the equilibrium situation, unit roots and genuine hysteresis are different in two aspects: (1) For unit root processes the memory is unselective, summing up every past shock while the memory of a (macro-) hysteretic system is selective, since only the non-dominated past extrema are resulting to a branch-to-branch transition (*Cross*, 1994). (2) Equal but opposite shocks will in the case of linear unit roots leave the equilibrium unchanged, whereas under hysteresis two successive shocks with opposite sign result in a new equilibrium and a remanence effect in a non-linear way (*Amable et al.*, 1994, and *Piscitelli et al.*, 2000, p. 59 f.).

Unit root dynamics in a multivariate system

Of course, unit root dynamics can be generalized to multivariate systems of difference equations. We use a standard error correction model as an example (see *Göcke*, 2002). As a first equation we use eq. (9) again. In our example Y could be national income which exposed to an exogenous shock R. As a second equation, we assume an error correction mechanism for the aggregate consumption C: In the long run, consumption C is proportional to income Y (with a ratio ‘c’, called the “propensity to consume”). Short-run deviations from this equilibrium relation $C=c\cdot Y$ are adjusted in the following period by a ratio ‘d’. Consumption is affected by another exogenous shock Z. The error correction system is:

$$\begin{aligned} &(12)\quad Y_t = b\cdot Y_{t-1} + R_t && \Rightarrow\ (1-b\cdot L)\cdot Y_t = R_t \\ & \Delta C_t = d\cdot(C_{t-1} - c\cdot Y_{t-1}) + Z_t && \Rightarrow\ d\cdot c\cdot L\cdot Y_t + [1-(1+d)\cdot L]\cdot C_t = Z_t \end{aligned}$$

with: $0 < b \le 1$, $-2 \le d \le 0$ and $0 \le c \le 1$

Using matrix and lag-operator representation, $A(L)\cdot y_t = r_t$, the system can be restated:

$$(13)\quad \begin{bmatrix} 1-b\cdot L & 0 \\ d\cdot c\cdot L & 1-(1+d)\cdot L \end{bmatrix}\cdot\begin{bmatrix} Y_t \\ C_t \end{bmatrix} = \begin{bmatrix} R_t \\ Z_t \end{bmatrix}$$

The characteristic polynomial of the lag-operator matrix A(L) and its roots z_1 and z_2 are:

$$(14)\quad \det(A(L)) = (1-b\cdot L)\cdot[1-(1+d)\cdot L] = 0 \quad\Rightarrow\quad z_1 = \frac{1}{b} \text{ and } z_2 = \frac{1}{1+d}$$

In the case of $b=1$ and $d=0$ unit roots ($|z_i|=1$) result. If the first equation shows a unit root ($b=1$) – while the second equation has a root outside the unit circle ($-2<d<0$), the whole system shows indifference and path-dependence concerning the levels of Y_t and C_t (i.e. these variables are integrated of order one), however, the relation between both ($C_t=c\cdot Y_t$) acts as a stable attractor (see *Amable et al.*, 1992, Fig. 7 and 9, for an illustration). This situation is called “cointegration” of C_t and Y_t. In such a situation, a deviation of the relation $C_t = c\cdot Y_t$ is reduced by an adjustment process, and $C=c\cdot Y$ can be seen as a long-run equilibrium. Therefore, the error correction concept is equivalent to cointegration between the variables of stochastic processes in time series analysis, as shown by *Engle, Granger* (1987). (For an introduction to cointegration see *Engle, Granger*, 1991, and *Lütkepohl*, 1991, pp. 346 ff.).

However, “unit root hysteresis” is a borderline case since the adjustment process degenerates to indifference only if the roots meet the complex unit circle *exactly* (*Amable et al.*, 1992, p. 14). For unit roots only near to the unit circle either a slow dynamic away from an unstable equilibrium or a gradual adjustment towards a definite stable equilibrium will result. Since economic time series are relatively short compared to the duration of economic adjustment processes, and since economic variables are consecutively exposed to external disturbances, an empirically significant differentiation between unit roots dynamics and situations close to a unit root is often impossible and statistical procedures testing on unit roots show very little power in differentiating between both cases (*Dickey, Fuller*, 1979).

(Typically in labor economics) cointegration models were extended by potential structural breaks of the (long-run) equilibrium relations in the sense of *Perron* (1989) and *Bai, Perron* (1998). However, the introduction of structural breaks in the cointegration/equilibrium relation leads to a rejection of degenerated unit roots adjustments in a lot of cases where unit roots could not be significantly rejected before explicitly considering structural breaks. If a unit root is rejected after extending the model by structural breaks, this is (by these authors) interpreted as a rejection of "hysteresis", since path-dependence based on unit-roots is seen as the main characteristic of hysteresis. In contrast, the structural breaks are in a so called "structuralist" view à la *Phelps* (1994) thought as reflecting changes in unobserved variables (e.g. of institutions, labor law etc.) resulting in a shift of the "non-accelerating rate of unemployment" (NAIRU) as a (for a while) stable equilibrium. Shifts in the equilibrium rate of unemployment are not seen as hysteresis, but only indifferent unit-root adjustment dynamics are (see e.g. *Ayala et al.*, 2012, *Liu et al.*, 2012, *Kanalıcı-Akay et al.*, 2011, as recent examples for this kind of (mis-)interpretation). From the point of view of "genuine" hysteresis (as presented in this chapter), this unit-root simplification falls short of the complex dynamics including branch-to-branch transitions between locally stable equilibria, i.e. path-dependent structural breaks. Thus, structural breaks are not counter evidence but rather a consequence of genuine hysteresis.

3.1.2 Non-linear time series models

Nowadays, time series econometricians usually apply non-linear techniques in order to test for hysteretic effects in economic relationships. *O'Shaughnessy* (2011), for example, provides an overview with regard to hysteresis in unemployment. However, *Hughes Hallet, Piscitelli* (2002) state that it is a challenging task to find hysteretic behavior in economic data because no explicit test for it exists.

Generally, it is possible to distinguish between several forms of non-linearity in time series models. On the one hand, there are authors modeling asymmetric adjustment to a long-run, i.e. cointegrating relationship (see, for example *Kannebly*, 2008). This is generally referred to as threshold cointegration. On the other hand, *Verheyen* (2013) applies the non-linear autoregressive distributed lags approach (NARDL) of *Shin et al.* (2011) to export demand and tries to derive stylized facts of export demand which are in accordance with hysteresis.

One problem of conventional linear cointegration models is the fact that these assume a linear error correction mechanism. However, various reasons exist why this adjustment might be non-linear. Just think of menu costs which imply that adjustment occurs only if a certain threshold value is surpassed. Otherwise, within this certain band of menu or transaction costs no adjustment occurs. Furthermore, strategic behavior could lead to different adjustment patterns depending on whether the deviation from the long-run equilibrium is positive or negative. If these arguments hold, conventional residual-based cointegration tests as the well-known *Engle, Granger* (1987) approach might fail to detect cointegration although a stable long-run equilibrium relationship exists.

In this vein, *Kannebley* (2008) applies the threshold cointegration model introduced by *Balke, Fomby* (1997) to Brazilian exports. He finds evidence in favor of non-linear behavior in nine out of 16 models for which he is able to detect a cointegrating relationship. According to his results adjustment to equilibrium takes place faster in cases when the deviations were below a certain threshold value. Relating these results to the trade hysteresis model of *Belke, Göcke* (2001, 2005), *Kannebley* (2008) concludes that market exit costs might be higher than market entry costs.

While threshold cointegration models incorporate non-linearities as a discrete regime shift smooth transition (STR) models popularized by *Teräsvirta* (1994, 1998) do not assume an abrupt switch between different regimes but rather allow for a gradual move from one regime to another and thus, allow so to speak for a continuum of regimes. Such a pattern is probably more plausible on a macroeconomic level as different firms might face different market entry and exit costs. In such a case, aggregation of individual firms would lead to no certain threshold value that triggers strong reactions as outlined in the previous sections but to a path with moderate and strong reactions. In STR models, system dynamics change according to a transition variable which might be an observable economic quantity. In case of trade relationships this could be for example the exchange rate. Model dynamics could differ between cases of currency appreciations or depreciations. For example, *O'Shaughnessy* (2001) applies such a framework to UK unemployment and capacity data.

Recently, *Verheyen* (2013) applied the NARDL approach of *Shin et al.* (2011) to export demand equations in order to derive some stylized facts of trade relationships. He especially focuses on possible non-linear exchange rate effects which should exist if hysteresis is present in export activity as shown in the previous sections. The NARDL approach proposes to split up a regressor into its partial sums and incorporate these series in the regression function.

Verheyen (2013) does so and incorporates partial sums of exchange rate series into export demand equations. Precisely he distinguishes two cases. Firstly, he differentiates between cases of currency appreciations and depreciations. His results indicate that exports react stronger to currency depreciations than to appreciations, which is an indication of strategic behavior of exporting firms in the sense that they might adjust their margins in order to cushion unfavorable price developments. Secondly, in order to test for hysteretic effects, he distinguishes between three cases when calculating the partial sums: large appreciations, small changes in exchange rates and large depreciations. As the sunk cost hysteresis model outlines large changes in exchange rates should lead to stronger reactions in exports than moderate changes. However, his results do not strongly correspond to such a pattern. One reason for this finding might be that the changes in exchange rates he considers might still be too small for hysteretic effects to occur.

Applying the NARDL method and using a 4-fold decomposition (i.e. small or large appreciations vs. small or large depreciations) *Fedoseeva (2013)* shows for export prices of German sugar confectionary that the pricing-to-market reactions of German exporters are neither symmetric nor linear.

One obvious reason which makes it difficult to find hysteretic effects in macroeconomic data is of course that the sunk cost hysteresis model is a microeconomic phenomenon. After aggregation, then, on a macroeconomic level various phenomena of non-linearity could look like hysteresis.

3.2 Models based on "strong" hysteresis

Now, two different methods to integrate "strong"/aggregate hysteresis-loops in empirical models are presented: (1) the *Piscitelli et al.* (2000) method as a direct application of the Preisach-dynamics and (2) the *Belke, Göcke* (1999a) play-algorithm. Both methods are based on calculating artificial "hysteresis variables" from the original forcing variables in order to integrate path-dependent multibranch-effects.

3.2.1 An algorithm describing Preisach-dynamics

Piscitelli et al. (1999, 2000) developed an algorithm to transform the forcing variables of an economic relation in a way that the Preisach-aggregation method is reproduced. Based on an assumption of the distribution of the α_j/β_j-set of entry and exit triggers of the heterogeneous firms and on the weight of these firms, the weight of the S_t^+-area of active firms (as outlined with Fig. 3) can be calculated. In a first step, the series of non-dominated extrema has to be identified and then the changes of the S_t^+-area with changes in the forcing variable are calculated. Usually, the most simple assumption of a uniform weight/distribution is applied, and thus only the acreage of triangles and quadrangles (i.e. quadratic functions) has to be computed in order to describe the aggregate loop (as depicted in Fig. 4). The *Piscitelli*-algorithm is e.g. applied by *Mota, Vasconcelos* (2012) to estimate hysteresis in the Portuguese labor market. A graphical representation of this transformation would look like the aggregate macro-loop in Fig. 4, if the original variable was depicted on the abscissa and its transformed hysteresis variable on the ordinate.

A disadvantage of the Piscitelli-algorithm is the distribution assumption of the α_j/β_j-sets representing firms, since this distribution determinates the curvature of the aggregate loop. However, as shown by *Hughes Hallet, Piscitelli* (2002) via simulation calculations, the results are quite stable concerning this simplifying uniform weight distribution assumption. Nevertheless, a situation with increasing uncertainty, will induce a widening of the band of inaction of non-ideal relays, i.e. an increase in the distance of the α_j/β_j-points from the 45°-($\alpha=\beta$)-line. The consequences of these dynamics in the α_j/β_j-distribution could not be addressed with this method, based on a static distribution of the α_j/β_j-sets.

3.2.2 The linearized play-model

Belke, Göcke (2001, 2005) as an advancement of *Göcke* (1994, 2001) develop an algorithm based on a kind of macroeconomic/generalized play-dynamics in order to approximate macroeconomic “strong” hysteresis-loops.[3] They assume linear partial segments with two different slopes: a play-section with a weak reaction (not necessary with a zero slope) and so called “spurt” sections with a stronger reaction of the dependent variable on changes of the forcing variable, i.e. with a steeper slope.[4] Fig. 5 gives an example: Start with point A (price p_0) located on the downward leading (left) spurt line. Now, the price changes the direction and increases, entering the play area and resulting in a weak play reaction results (point B) until the entire play area is passed (p_1, point C). A further increase to p_2 induces a strong response of y along the (right) upward leading spurt line (point D). A following price decrease first (up to p_3) takes place on a new play-line DE, which is shifted upwards and to the right compared to the “old” play-line AC. A further price decrease (to p_4, point F) then runs on the left spurt-down-line. Again, the play area is shifted, now downwards and to the left (to line FG) by a past spurt movement. Summarising, persistent effects result, if movements go beyond a play area and take place on a spurt-line: As in the standard case of play-hysteresis, there are no permanent effects from small variations taking place only inside the play-area.

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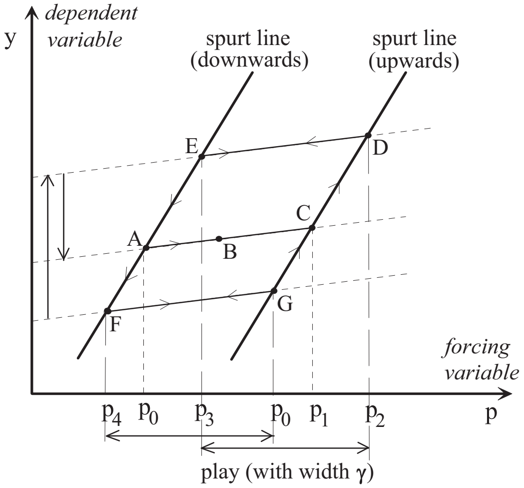
[3] For an application of the play-model see *Mota, Varejão, Vasconcelos* (2012).

[4] See *Pindyck* (1988), pp. 980 f., *Dixit, Pindyck* (1994), pp. 15 f., for “spurts” based on a microeconomic sunk cost mechanism.

Fig. 5 – Linear play-hysteresis and spurt areas



Based on summing up the p-movements on the spurt-line the *Belke/Göcke*-algorithm calculates an artificial shift variable. This "*spurt variable*" s_t integrates all spurt movements which had led to shifts of the play area. The spurt variable s_t is just the series of the original forcing variable p_t where all small movements inside the play areas are filtered out. Since filtering is based on the play width γ, the resulting spurt variable s_t depends on the size of γ. As a result a standard linear equation of the following type has to be estimated (e.g. by OLS):

(15) $y_t = \text{constant} + \alpha \cdot p_t + \beta \cdot s_t(\gamma) + \text{function(further variables)}$

The coefficient α is the low slope inside the play areas, and the coefficient β of the filtered series s_j is the *difference in slope* between the weak play and the strong spurt reaction. The complex path-dependent dynamics are captured in a simple linearized way, by adding only two new parameters: (1) the width of the play area γ, and (2) the slope difference β of spurt sections compared to play sections.

Since uncertainty results in a widening of the band of inaction on a micro-level, increased macroeconomic uncertainty will result in a shift of all α/β-points away from the α=β-45°-line. This would result in a widening of the play area on the aggregate level. The algorithm was extended to allow for such effects (see *Belke, Göcke*, 2005, for a version with a variable play-width related to varying uncertainty).

3.3 An example for estimating play-hysteresis

In this section, we present an example of estimating play-hysteresis in German exports to Japan, in analogy to *Belke et al.* (2013). The data are mainly from Eurostat: We use the quarterly export value in euros from 1991 to 2012. The regarding exchange rates are quarterly averages of monthly averages reported by Deutsche Bundesbank (before 2000 converted DM data). GDP-deflators were used for calculating real exports and the real exchange rate. As additional explanatory variables we include the real GDP of Japan as the destination country (one quarter lagged and seasonally adjusted), a linear trend, and 3 quarterly dummies (Q1, Q2, Q3). Eq. (15) is estimated with the FM-OLS method in the following form,

$$\mathrm{EXP}_t = \mathrm{CONSTANT} + \alpha \cdot \mathrm{ER}_t + \beta \cdot \mathrm{SPURT}(\gamma) + \mathrm{GDP}_{t-1} + \mathrm{TREND} + \mathrm{Q1} + \mathrm{Q2} + \mathrm{Q3},$$

where EXP_t is the real export value (from Germany to Japan) and ER_t is the real exchange rate in indirect quotation. The spurt variable was calculated based on filtering ER as the original variable (using play width γ) and is represented by $S(\gamma) = \mathrm{SPURT}(\gamma)$. In a first step – for reasons of comparison - we estimated eq. (15) without the non-linear spurt component to get the standard/linear model and received the following regression results (using the software-package EViews):

Tab. 1: Linear standard regression

```
Dependent Variable: EXP
Method: Fully Modified Least Squares (FMOLS)
Sample: 1995Q1 2012Q4
Included observations: 72
Cointegrating equation deterministics: CONSTANT TREND Q1 Q2 Q3
===========================================================================
       Variable          Coefficient     Std. Error    t-Statistic   Prob.
===========================================================================
       CONSTANT            -6372.275      1778.814     -3.582317    0.0007
          ER           α = -18.17429      2.602108     -6.984448    0.0000
    JAPAN_GDP(-1)           0.009061      0.001651      5.486520    0.0000
        TREND              -0.569324      4.181815     -0.136143    0.8921
          Q1               -25.05944      102.4723     -0.244548    0.8076
          Q2               -225.0373      102.5192     -2.195075    0.0317
          Q3               -17.68709      102.3520     -0.172806    0.8633
===========================================================================
R-squared                   0.741734   Mean dependent var           3066.994
Adjusted R-squared          0.717894   S.D. dependent var           494.6592
S.E. of regression          262.7313   Sum squared resid            4486804.
Durbin-Watson stat          0.897385   Long-run variance            94212.79
===========================================================================
```

In order to estimate the optimal play width, we run a grid search for different potential values of the play width γ – and choose the γ-value which results in the spurt-regression with the highest R^2. The grid search referring to the R^2-level for different γ-values and the estimated value of $\gamma = 17.25$ is represented in Fig. 6. The dynamics of the exchange rate and of the spurt variable - where movements inside the play are filtered out - are depicted in Fig. 7.

Fig. 6 – Grid-Search for different play-widths γ

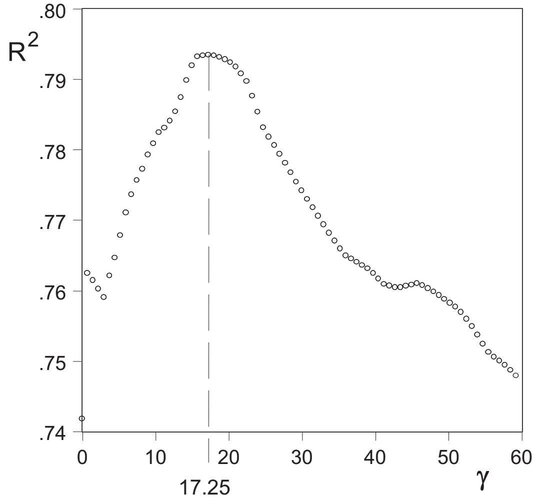
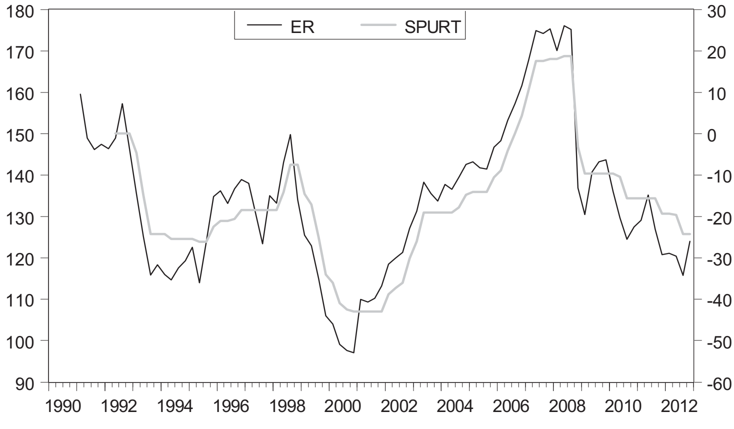


Fig. 7 – Exchange rate and resulting spurt variable (play width $\gamma = 17.25$)



The result of the non-linear play/spurt-regression using this optimal play width ($\gamma = 17.25$) is as follows:

Tab. 2: Non-linear play/spurt-regression

Dependent Variable: EXP
Method: Fully Modified Least Squares (FMOLS)
Sample: 1995Q1 2012Q4
Included observations: 72
Cointegrating equation deterministics: CONSTANT TREND Q1 Q2 Q3

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CONSTANT	-8632.354	2054.556	-4.201567	0.0001
ER	α = -5.269178	5.644607	-0.933489	0.3541
SPURT	β = -15.83624	7.002543	-2.261499	0.0271
JP_GDP(-1)	0.009233	0.001669	5.533511	0.0000
TREND	0.107063	4.238531	0.025259	0.9799
Q1	-44.70860	103.1600	-0.433391	0.6662
Q2	-221.6490	103.2352	-2.147029	0.0356
Q3	-15.01675	103.0401	-0.145737	0.8846

R-squared	0.793414	Mean dependent var	3066.994
Adjusted R-squared	0.770818	S.D. dependent var	494.6592
S.E. of regression	236.8077	Sum squared resid	3588986.
Durbin-Watson stat	0.984079	Long-run variance	95446.87

If the coefficient β of the spurt variable is significant, this is an indication of non-linear play-hysteresis. As the exchange rate is defined in indirect quotation, we expect (and received in both estimations) a negative coefficient of the exchange rate and of the spurt – as an appreciation of the exporter's currency usually leads to a reduction of exports. In case of the non-linear model the sum of the coefficients of the exchange rate and the spurt variable (α+β) should be negative and represent the stronger spurt reaction, while the coefficient α now represents a weak play reaction. Thus, compared to the linear standard regression, the coefficient α of the original variable (ER) should be smaller in size (or even insignificant) if the filtered spurt variable is added in the non-linear play-model. A comparison of both regressions (linear standard model versus non-linear play-model) shows, that the inclusion of non-linear play-dynamics improves the fit (indicated by an increase of the adjusted R^2 and by the t-value of the spurt variable). Furthermore, the theoretical assumptions about the direction and the size of play- versus spurt-reactions are corroborated, if the estimation results for the α- and β-coefficients are compared.

4. Conclusion

This contribution has provided an overview of the phenomenon of hysteresis in economics. Especially unemployment dynamics and export activity can show signs of hysteresis as outlined in the previous sections. Hysteresis generally refers to a situation in which there is a permanent change in a quantity that is triggered by some other variable which has changed only temporarily. Thus, the causal effect remains although the initial cause has vanished.

In economics, adjustment costs can result in hysteretic behavior. The microeconomic model presented in Section 2 has shown that due to market entry and exit costs which are sunk ex post non-linear patterns of a firm's market activity could arise. Additionally, we have sketched that one has to distinguish between hysteresis on a micro, i.e. firm level and an aggregated macroeconomic scale.

Furthermore, Section 3 has presented various econometric approaches that try to identify hysteresis in economic relationships. While this is generally a challenging task – as there is no standard formal tests of economic hysteresis – different empirical approaches have found patterns that are in accordance with this phenomenon. Nevertheless, progress in non-linear time series modeling and panel econometrics should provide better tools to model hysteretic behavior more adequately.

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