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Christian Dudel, Jan Marvin Garbuszus, and Julian Schmied[1]

Assessing Differences in Household Needs: A Comparison of Approaches for the Estimation of Equivalence Scales Using German Expenditure Data

Abstract

Equivalence scales are routinely applied to adjust the income of households of different size and composition. Because of their practical importance for the measurement of inequality and poverty, a large number of methods for the estimation of equivalence scales have been proposed. Until now, no comprehensive comparison of these methods has been conducted. In this paper, we employ German household expenditure data to estimate exact equivalence scales using several parametric, semiparametric, and nonparametric approaches. Using a single dataset, we find that the resulting equivalence scales do not differ greatly by method; with the exception of some outliers that yield implausible scales. Most of the equivalence scales we estimate are close to the modified OECD scale. Calculating the Gini coefficient, we find that the at-risk-of-poverty rate and the interquartile range based on the more plausible equivalence scales lead to relatively consistent assessments of inequality and poverty. We conclude that differences in estimation methods for equivalence scales might be less important than was previously thought.

JEL Classification: D12, I32, C14, C21, C31

Keywords: Equivalence scales; household demand; inequality measurement; equivalence scale exactness; Engel curves; independence of base

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1 Introduction

Equivalence scales are used to make the incomes of households of different sizes and compositions comparable. A well-known example of an equivalence scale is the so-called modified OECD scale (Hagenaars et al., 1994). The household of an adult living alone is used as a reference, and is assigned a value of one. Adding individuals aged 14 and older to the household increases this value by 0.5 per person, and adding children below age 14 increases it by 0.3 per child. Thus, for instance, a household of two adults with one child has an equivalence scale value of 1.8. Dividing the income of such households by 1.8 gives an equivalence income (or equivalized income), which is standardized relative to the reference household, and can be directly compared across household types.

Because they can easily be applied, the modified OECD scale and other equivalence scales are widely used in applied research. Despite their popularity, several drawbacks of using equivalence scales have been pointed out in the literature (e.g., Lewbel, 1989b; Blundell and Lewbel, 1991). Among the practical issues that have been mentioned are that the results of inequality and poverty measurements depend to a large extend on the specific equivalence scale used, and that a wide range of equivalence scales can be applied (e.g., Buhmann et al., 1988; Szelky et al., 2004). The main reason why there are so many different equivalence scales is that a wide range of methods have been proposed for estimating equivalence scales, and there is no clear consensus on which method should be used (Phipps and Garner, 1994; Muellbauer and van de Ven, 2004). Comparisons of methods for the estimation of equivalence scales are surprisingly scarce in the literature, and the existing comparisons have covered only few approaches (e.g., Lancaster and Ray, 1998).

In this paper, we conduct a direct comparison of several different methods for the estimation of equivalence scales using the same dataset. Employing data from the German Sample Survey of Income and Expenditure (*Einkommens- und Verbrauchsstrichprobe*; EVS), we focus on approaches that estimate a single equivalence scale value per household based on expenditure data. We then use the resulting equivalence scales to calculate indices of inequality and poverty. We include in our analysis at least one representative of each of the most important types of expenditure-based methods. The main types of methods are the classic single-equation approaches (Engel's approach, see Deaton and Muellbauer, 1986); the Linear Expenditure System (LES; Stone, 1954) and some of its variants (Lluch, 1973; Howe et al., 1979; Pollak and Wales, 1980), which have often been applied to German expenditure data; the Almost-Ideal Demand System (Deaton and Muellbauer, 1980) and its quadratic extension (Banks et al., 1997), which are now the standard approaches used for modeling household demand; semiparametric approaches

(Pendakur, 1999; Stengos et al., 2006); and a completely nonparametric approach based on the counterfactual framework (Dudel, 2015). These methods roughly span a continuum in terms of model complexity, data requirements, and the restrictiveness of the underlying assumptions.

We find that when exactly the same dataset is used, the equivalence scale estimates vary only slightly; with the exception of some outliers that give implausible results. Given the narrow range of these equivalence scales, it is not surprising that the estimates of the Gini coefficient, the at-risk-of-poverty ratio (ARP), and the interquartile range (IQR) are also not shown to vary widely.

Our paper contributes to the literature in several ways. First, to the best of our knowledge, we are conducting the first comparison of methods for the estimation of equivalence scales that covers more recent methodological developments from the literature, and that uses recent data. Second, our findings show that while equivalence scales differ considerably, a subset of the approaches in our application lead to plausible equivalence scales and to consistent results with respect to inequality and poverty measurements. Third, the existing overviews of equivalence scales tend to obscure the differences between the methods applied because the countries, the datasets, and the time periods used in conjunctions with these methods vary. For instance, equivalence scale estimates for several different countries are often shown next to each other. While some countries have similar scales (Phipps and Garner, 1994; Burkhauser et al., 1996), this is not always the case, and discrepancies are possible (Lancaster et al., 1999). Similar issues might arise for equivalence scales based on different datasets because, for example, of differences in the variables used or in the preparation of the data (Dudel et al., 2017a); and for equivalence scales estimated for different points in time because, for example, of price changes (Pendakur, 2002). Our analysis avoids these issues. Our findings demonstrate that a large share of the variation in exact equivalence scales found in the literature is indeed attributable to such factors, and that a smaller share of the variation can be explained by the use of different models.

The remainder of this paper is structured as follows. The basic assumptions of equivalence scales are introduced in section 2. The approaches we apply to estimate equivalence scales are explained in section 3. The dataset we use and the subset selection process are described in section 4. In section 5, we give the results for the equivalence scales and the resulting estimates of poverty and inequality. Section 6 concludes.

2 Equivalence scales and identification

2.1 Preliminaries and basic definition of equivalence scales

Let $\mathbf{z} = (z_1, \ldots, z_k)$ denote a vector of k household characteristics such as household size, number of children, or age of household members. All households can choose between m goods with prices captured in a vector $\mathbf{p} = (p_1, \ldots, p_m)$. Household demand is given by the demand function $D(p, x, z) = \mathbf{q} = (q_1, \ldots, q_m)$, where q_i is the demand for good i and x is household income. Furthermore, household utility is given by $U(\mathbf{q}, z)$.[1] Finally, an expenditure function can be defined by $E(u, \mathbf{p}, \mathbf{z}) = \min_q[\mathbf{p'q}|U(\mathbf{q}, z) = u]$.

Using these preliminaries, household equivalence scales are defined as

$$S(u, \mathbf{p}, \mathbf{z}_h, \mathbf{z}_r) = \frac{E(u, \mathbf{p}, \mathbf{z}_h)}{E(u, \mathbf{p}, \mathbf{z}_r)}, \quad (1)$$

where $\mathbf{z}_h$ and $\mathbf{z}_r$ are the household characteristics of two different households h and r. Thus, an equivalence scale is a function that returns the ratio of the expenditures of two households of different compositions with the same level of utility and facing the same prices. The reference household $\mathbf{z}_r$ is usually fixed as the household of a single adult, but any other household type could also be chosen. We assume the former type, and will write $S(u, \mathbf{p}, \mathbf{z}_h)$.

2.2 Identification and income-independence

Equivalence scales as defined by equation (1) are not identified if ordinal utility is assumed (Pollak and Wales, 1979; Lewbel, 1989a; Blundell and Lewbel, 1991; Pollak, 1991). This is because equivalence scales require interpersonal comparisons of utility that are not possible assuming ordinal utility. Any approach for the estimation of equivalence scales has to deal with this issue of identification.[2]

In expenditure-based approaches, a common solution to the identification problem is to employ (indirect) utility functions of a certain structure. For instance, if we assume that equivalence scales do not depend on the welfare level — i.e., $S(u, \mathbf{p}, \mathbf{z}_h) = S(\mathbf{p}, \mathbf{z}_h)$

[1]Note that household utility functions typically ignore the distribution of resources within the household, and may thus be hard to defend, as it is individual household members who derive utility from consumption (Phipps and Burton, 1995). Still, household utility functions are the theoretical foundation of equivalence scales, and including individual needs and preferences and intrafamily bargaining in the derivation of equivalence scales is beyond the scope of this paper.

[2]Here, we will focus on approaches based on expenditure data. Other approaches not covered in this paper are expert-based scales and so-called subjective scales; Coulter et al. (1992). For a review of expert-based scales, see Fisher (2007); and for a review of subjective equivalence scales, see Schröder (2004).

— they can be identified (e.g., Blundell and Lewbel, 1991). This assumption is called, or is related to, *independence of base* (Lewbel, 1989b) and *equivalence scale exactness* (Blackorby and Donaldson, 1993). For practical purposes, this assumption often — but not always — implies that equivalence scales do not depend on the expenditure levels (or income levels) of the households under consideration. More specifically, equivalence scales are income-independent if the same value is applied to all households of a certain type. While some of the methods we use do not require this assumption, all of the estimates we present are income-independent in the sense that we do not differentiate by income level.[3]

3 Expenditure-based methods for the estimation of equivalence scales

3.1 Engel's approach

The idea of using household expenditures to assess household welfare is usually attributed to Engel (1895), and is based on the observation that the share of household expenditures spent on food depends on household type, and declines as income rises. Assuming that two households achieve the same level of welfare if the share of their expenditures allocated to food are equal, equivalence scales can be identified by comparing the incomes of different types of households that allocate the same share of their expenditure to food.

This approach can be implemented as follows (Deaton and Muellbauer, 1986). To simplify notation and without loss of generality, we assume that we are only interested in comparing two household types: namely, the reference household type and some other comparison household type. Letting w_f denote the share of expenditures on food, the following regression equation, as proposed by Working (1943), can be estimated based on demand data (also see Leser, 1963):

$$w_f = \alpha + \beta_x \log(x) + \beta_h d_h + \gamma' \mathbf{z}, \tag{2}$$

where x is household income, d_h is a dummy variable capturing whether households are of the comparison type ($d_h = 1$) or are reference households ($d_h = 0$), and $\mathbf{z}$ captures socio-demographic variables other than household type. Now consider two households that

[3]In recent years, approaches have been proposed that relax the *independence of base* assumption (e.g., Donaldson and Pendakur, 2004, 2006). While several recent studies have supported the idea of decreasing scales (e.g., Koulovatianos et al., 2005; Biewen and Juhasz, 2017), this approach has not been broadly applied in welfare analysis and poverty research. Therefore, we restrict ourselves to scales that provide only a single equivalence value per household.

allocate the same share of their expenditures to food as given by equation (2), but that are of different types. Equating expenditure shares and solving for the ratio of incomes x_h and x_r that the households need to achieve the share spent on food gives

$$S = \frac{x_h}{x_r} = \exp\left(-\frac{\beta_h}{\beta_x}\right). \tag{3}$$

This approach assumes that equivalence scales do not depend on income or expenditure levels. Moreover, prices are usually not included, even though it would be possible to do so. This approach therefore has low data requirements, and it is easy to apply. Engel curves, as defined by (2), are linear. Linear Engel curves are not necessary for this approach (Leser, 1963), but empirical applications nevertheless typically use linear Engel curves.

3.2 Linear Expenditure System and extensions

The Linear Expenditure System (LES) proposed by Stone (1954) is the earliest full demand system; meaning that it is based not on a single equation, but a system of equations, each of which covers expenditures for one of the m goods. Starting from a Stone-Geary utility function, the following set of m expenditure functions can be derived as follows:

$$\begin{array}{rcl} y_1 & = & p_1 a_1 + b_1 \left(x - \sum_{j=1}^{m} p_j a_j\right) \\ & \vdots & \\ y_i & = & p_i a_i + b_i \left(x - \sum_{j=1}^{m} p_j a_j\right) \\ & \vdots & \\ y_m & = & p_m a_m + b_m \left(x - \sum_{j=1}^{m} p_j a_j\right) \end{array}, \tag{4}$$

with $y_i = p_i q_i$, i.e., expenditure for good i; $p_i a_i$ being interpreted as the minimum expenditure on good i; and b_i being the marginal budget share of good i, with the restriction that $\sum b_i = 1$.

This set of equations can be estimated separately for each household type (for an estimation of the LES, see, e.g., Deaton, 1975). Given these parameter estimates, a pragmatic way to calculate equivalence scales is based on a comparison of the minimum expenditures by household type (e.g., Kohn and Missong, 2003),

$$S = \frac{\sum_{i=1}^{m} a_i^h}{\sum_{i=1}^{m} a_i^r}, \tag{5}$$

where a_i^r is the minimum expenditure on good i of the reference household; and a_i^h is the minimum expenditure of a comparison household.

The LES has inspired several extensions, of which we cover three variants: the Extended Linear Expenditure System (ELES; Lluch, 1973), the Quadratic Expenditure System (QES; Howe et al., 1979), and the Functionalized Extended Linear Expenditure System (FELES; Merz, 1983). Essentially, the ELES expands the LES by introducing saving, which is treated as an additional commodity; the QES assumes a quadratic relationship between expenditure and (marginal) income, in contrast to the linear Engel curves of the LES; and the FELES allows us to let minimum expenditure a_i and marginal expenditure shares b_i depend on socio-demographic variables, without the need for separate estimation by household type. For all of these variants, the equivalence scale can be calculated in the same way as in the basic LES. In addition we include a scaled QES as proposed by Howe et al. (1979).

In terms of data demand, the LES and its extensions fall somewhere in the middle: expenditure data are needed for several expenditure categories; whereas data on prices can be included, but are not needed, as p_i can be set to one. All of the variants are income-independent and are based on the independence of base assumption; although the QES uses quadratic Engel curves.

3.3 Almost-Ideal Demand System and extensions

The LES and its extensions have been used extensively to analyze household demand and to estimate equivalence scales, especially in the 1970s and 1980s. But in recent years, the LES and its extensions have mostly been replaced by the Almost-Ideal (AI) Demand System suggested by Deaton and Muellbauer (1980). The AI system arose from the search for a model that provides a good fit for empirical demand data, while having properties deemed desirable for demand systems. Starting from the price-independent generalized logarithmic (PIGLOG) class of preferences, the expenditure share for good i, w_i can be derived to equal

$$w_i = \alpha_i + \sum_{j=1}^{m} \gamma_{ij} \log p_j + \beta_i \log\left(\frac{x}{P}\right), \tag{6}$$

with

$$\log P = \alpha_0 + \sum_{i=1}^{m} \alpha_i \log p_i + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \gamma_{ij} \log p_i \log p_j.$$

γ_{ij} capturing the effect of the price of good j on the share of expenditures on good i, β_i being the marginal effect of log income, and α_i is a parameter. P is a price deflator for income. As P makes the model nonlinear, in empirical applications linear approximations are often used (see, e.g., Barnett and Seck, 2008). Here we will use the (nonlinear) translog price index, as proposed by Deaton and Muellbauer (1980).

To estimate equivalence scales, some parameters have to be added to the AI demand system. We follow a general approach suggested by Ray (1983) for introducing equivalence scales in demand systems. If we want to compare the reference household to one other household type only, this approach is implemented by using

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i^* \log \left(\frac{x}{SP} \right), \tag{7}$$

where

$$S = 1 + \rho d_h$$
$$\beta_i^* = \beta_i + \eta_i d_h$$

assuming that the comparison household needs more resources than the reference household. S denotes the equivalence scale value, and ρ captures the needs of the comparison households relative to the needs of the reference household. η_i plus β_i gives the income elasticity for the comparison household. Given P, the parameters can be found using nonlinear seemingly unrelated regressions (Greene, 2012).

Similar to the LES, the AI demand system essentially assumes that the relationship between log income and expenditure shares is linear. But for some commodities, this relationship has been found to be nonlinear. To account for the nonlinearity, and to provide a better fit for the demand data, Banks et al. (1997) introduced the Quadratic AI demand system. The QAI demand system essentially includes an additional quadratic term of (deflated) log income. Equivalence scales are estimated by expanding the approach of Ray (1983) to this term.

The AI and the QAI demand system are rather flexible models that can fit many patterns of household demand, but they also require data on prices. Thus unlike in Engel's approach and the LES, at least two cross-sections of demand data are required. Equivalence scale exactness is required, although the QAI demand system can be replaced with a less strict variant that allows for the estimation of income-dependent equivalence scales based on quadratic Engel curves.

3.4 Semiparametric approaches

The approaches presented so far all rely on the assumption that the relationship between log (deflated) income and expenditure or expenditure shares is linear or quadratic. While this assumption might be appropriate for some commodities, it might not hold for others (Banks et al., 1997). In an effort to address this problem, Pendakur (1999) developed a semiparametric approach to estimating equivalence scales that avoids strong assumptions regarding the relationship between income and expenditure shares by estimating nonlinear Engel curves. Writing the expenditure share for food, w_f, as a function of income y, prices p, and household type d_h, the approach assumes that

$$w_f(p, \log(y), d_h = 0) = w_f(p, \log(y) + \phi, d_h = 1) + \mu(p). \tag{8}$$

Here, the relationship between log income and the expenditure share for food as captured by $w_f(p, \log(y), d_h)$ can be of any functional form. It is, however, assumed that this functional form is equal across household types ("shape invariance"), and is only shifted vertically by price elasticity, $\mu(p)$, and horizontally by the log equivalence scale ϕ. Equivalence scales can be calculated as

$$S = \exp(\phi). \tag{9}$$

Estimation proceeds by using nonparametric methods to estimate the shape of $w_f(p, \log(y), d_h = 0)$ and of $w_f(p, \log(y), d_h = 1)$. In a second step, assuming constant prices, the log equivalence scale ϕ is found via a grid search, whereby the difference between the two sides of equation (8) is minimized (Pendakur, 1999). Stengos et al. (2006) proposed a variant of this method, which we also include in the set of methods we apply. They modified the second step of the approach, penalizing high or low values of ϕ. This yields more plausible estimates than the original method of Pendakur (1999). In particular, for comparisons where the income distributions of the reference and the comparison household types overlap slightly.

While the semiparametric approach is flexible regarding the functional form of Engel curves, it requires the independence of base assumption (Pendakur, 1999). The data requirements are relatively low, as a single cross-section of data suffices. In principle the share of expenditures on food can be replaced with the share of expenditures on other commodities. For instance, it would be possible to implement the ideas of Rothbarth (1943) in a semiparametric way. A drawback of the semiparametric approach is that including covariates in the first estimation step is not straight forward. Moreover, the approach relies to some extent on the selection of homogenous subsets of households

(Dudel et al., 2017b).

3.5 Counterfactual approaches

The counterfactual approach rephrases equivalence scales in the potential outcomes framework (e.g., Holland, 1986). Let us assume that in theory, every household can be considered as belonging to the reference household type (e.g., childless couple) and the comparison household type (e.g., couple with one child). $y^0(u)$ is the income needed to achieve utility u when the household is of the reference type, and $y^1(u)$ is the income needed to achieve utilty u when the household is of the comparison type. Assuming that a household achieves utility level u^0 when it is of the reference type, the equivalence scales are given by $y^1(u^0)/y^0(u^0)$ (Szulc, 2009; Dudel, 2015).

In practice, either $y^0(u)$ or $y^1(u)$ is observed; never both. That is, at any point in time, some households are observed as being of the reference type, but not of the comparison type, and vice versa. Still, under some assumptions, the marginal distributions of $y^0(u)$ or $y^1(u)$ can be estimated (e.g., Imbens, 2006). However, this approach is not sufficient for estimating estimate equivalence scales. Taking expectations, we have

$$\mathrm{E}\left[\frac{y^1(u^0)}{y^0(u^0)}\right] = \frac{\mathrm{E}\left[y^1(u^0)\right]}{\mathrm{E}\left[y^0(u^0)\right]} - \frac{1}{\mathrm{E}\left[y^0(u^0)\right]}\mathrm{Cov}\left[\frac{y^1(u^0)}{y^0(u^0)}, y^0(u^0)\right]. \tag{10}$$

The covariance term on the right-hand side requires the joint distribution of $y^0(u)$ and $y^1(u)$, which is not point identified (Abbring and Heckman, 2007). Szulc (2009) avoids this problem by estimating the geometric mean of $y^1(u^0)/y^0(u^0)$ instead of (10), while Dudel (2015) has proposed lower and upper bounds on (10). That is, the equivalence scales are not point-identified. For the comparison of, say, childless couples and couples with one child, the equivalence scales do not take on one specific value x, but can only be shown to be in an interval $[x^-, x^+]$. We adopt this partially identified approach here. Estimation proceeds using a nonparametric method suggested by Fan et al. (2017).

In contrast to previous approaches this identification strategy does not rely on the assumption that equivalence scales are independent of the welfare level. Further, it does not rely on any specific Engel curve shape. While this approach thus requires few assumptions, it does not allow us to achive any point estimates. Moreover, the interval estimates generated using this approach might not be informative if they are wide.

4 Data and implementation

4.1 Expenditure and price data

We applied the methods described in the previous section to data of the German Sample Survey of Income and Expenditure (*Einkommens- und Verbrauchsstichprobe*; EVS). The EVS is a quinquennial survey conducted by the German Federal Statistical Office that covers about 0.2% of households in Germany. We use data from the years 2003, 2008, and 2013. Each of the three cross-sections consists of more than $42,000$ households. For each household, detailed information on the household's income, expenditures, and savings is collected for one quarter of the year. In addition, socio-demographic variables are included, like the educational attainment of household members.

Expenditure information is collected based on a German equivalent of the United Nations' Classification of Individual Consumption According to Purpose (COICOP). Total expenditure is broken down into 12 commodity groups: (1) food and non-alcoholic beverages; (2) alcoholic beverages and tobacco; (3) clothing and footwear; (4) housing, water, electricity, and heating; (5) furniture, household equipment, and routine household maintenance; (6) health; (7) transportation; (8) communication; (9) recreation and culture; (10) education; (11) restaurants and hotels; and (12) miscellaneous goods and services. These expenditure categories are in turn based on more detailed expenditure information, but for estimation purposes we use these 12 categories. The information on income is also detailed, but we use aggregated net income; that is, income after taxes and social security contributions (for details, see Statistisches Bundesamt, 2013).

Price information for each of the 12 expenditure categories was also provided by the German Federal Statistical Office. Monthly prices were aggregated into quarterly prices by calculating the average. We thus include annual price variation between the years 2003, 2008, and 2013; as well as quarterly/seasonal variation within these years.

The socio-demographic variables we use include the number of adults and the number of children in a household. The household type is assigned based on these two variables. We distinguish between single adults (A); childless couples (AA); and couples with one child (AAC), with two children (AACC), and with three children (AACCC). Households of single adults are used as the reference household type for all equivalence scales.

Additional control variables are dummy variables indicating whether both partners in a couple are full-time employed; as well as dummy variables capturing the quarter of the year (spring, summer, autumn, winter), the age of the household head, the level of education (1 = no education, 2 = vocational training, 3 = foreman, 4 = college, 5 = university degree), and the type of residential area (ranging from one for rural areas to seven for densely populated areas in cities).

4.2 Preprocessing

From the pooled EVS data for the years 2003, 2008, and 2013 we have selected a homogeneous subset of households. This approach makes households of different types more comparable, and eases the interpretation of equivalence scale estimates. We have selected households in which at least one of the adults is below age 65, and at least one of the adults is receiving an income from being employed. We exclude pensioners because they are not of major interest when calculating equivalence scales for children. We also exclude households living only on welfare benefits, as our equivalence scales might otherwise be influenced by the equivalence scales implied by welfare benefits for different household types. Households with children above age 18 are also excluded.

We have further restricted the set of households to those residing in western Germany, as there are large economic differences between eastern and western Germany; Brenke and Zimmermann (2009). Moreover, we include only those households who rent or own their home, while excluding the small number of households living in accommodations paid for by an employer.

Table 1: Expenditure categories

	Monthly* expenditures in EURO				Expenditure shares in %				Change of prices (log)			
	Min	Mean	Sd	Max	Min	Mean	Sd	Max	Min	Mean	Sd	Max
Food	8.3	312.8	170.1	1128.0	0.4	13.1	5.2	55.0	4.6	4.7	0.0	4.7
Alcohol	0.0	38.2	48.7	302.0	0.0	1.7	2.3	21.2	4.6	4.6	0.1	4.7
Clothing	0.0	135.6	116.8	802.7	0.0	5.4	3.8	32.1	4.6	4.6	0.0	4.7
Housing	40.0	829.8	357.3	2465.0	5.5	35.9	10.8	89.5	4.6	4.6	0.0	4.7
Furniture	0.0	115.3	171.4	1529.0	0.0	4.2	5.3	51.5	4.6	4.6	0.0	4.6
Health	0.0	75.7	130.3	1216.7	0.0	2.8	4.2	41.9	4.6	4.6	0.0	4.6
Transportation	0.0	319.5	360.4	5046.7	0.0	12.2	8.8	83.0	4.6	4.6	0.0	4.7
Communication	0.0	69.9	37.3	266.0	0.0	3.2	1.9	22.4	4.5	4.6	0.1	4.7
Recreation	0.0	278.6	255.1	1907.7	0.0	10.8	7.4	56.3	4.6	4.6	0.0	4.7
Education	0.0	24.3	56.5	602.0	0.0	0.9	2.3	36.1	4.5	4.6	0.0	4.7
Restaurants	0.0	141.4	141.6	968.7	0.0	5.6	4.8	48.4	4.6	4.6	0.0	4.7
Miscellaneous	0.0	98.9	92.9	818.7	0.0	4.0	3.2	34.4	4.6	4.6	0.0	4.7

Note: The reporting period of the EVS denotes three months: the values shown are divided by three, and can therefore be regarded as approximately monthly.
Data: EVS 2003, 2008, 2013

Finally, we have also removed households from our analysis who reported extreme values of expenditures or income. First, we have removed households with a net income below the approximate welfare benefit level (*Arbeitslosengeld II*) for that type of household; where the welfare benefit threshold excludes housing costs. For instance, in 2013, welfare benefits were 382 EUR for a single adult, 690 EUR for a childless couple, and 224 EUR for additional children. Second, we have removed households with no expenditures

Table 2: Further descriptive statistics

Monthly* total expenditures (in EURO), mean	2,440
Monthly* net income (in EURO), mean	3,703
Monthly* net income (in EURO), min	382
Monthly* net income (in EURO), max	12,752
Age of the household head (in YEARS), mean	43
Share of single households (A, in PERCENT)	37
Share of couple households (AA in PERCENT)	29
Share of couple households with one child (AAC, in PERCENT)	29
Share of couple households with two children (AACC, in PERCENT)	29
Share of couple households with two children (AACCC, in PERCENT)	29
Share of tenures (in PERCENT)	48
Share of low educated (in PERCENT)	5
Share of higher educated (in PERCENT)	42
Share of people from low density areas (in PERCENT)	10
Share of dual earners (in PERCENT)	26

Note: * The reporting period of the EVS denotes three months: the values shown are divided by three, and can therefore be regarded as approximately monthly. ** Including persons with Fachabitur or Abitur. *** Including persons with no degree or a degree from a Hauptschule.
Data: EVS 2003, 2008, 2013

on food. Third, we have identified and removed outliers for each expenditure category and year. An outlier is defined as spending more than the median plus two and a half standard deviations on a commodity.

The final pooled sample we use for our analysis consists of $25,090$ households. Descriptive statistics for this sample are shown in Tables 1 and 2.

4.3 Implementation

For every model except the ELES a set of control variables is included. This set of control variables contains either the net income (for the Engel, the ELES and the FELES; semiparametric and nonlinear estimation) or the total expenditure. In addition, a set of dummy variables for education levels 1 - 6, observation quarters 1 - 4, and living regions 1 - 7 is included. The standard set is completed by the inclusion of two dummies indicating whether the household pays rent and is dual-income. For each dummy set the reference is set to one. For the Engel approach an additional dummy indicating whether the household has a large number of members is included. For the Almost-Ideal Demand System approaches, a set of dummies indicating the household type is included with the reference single adult.

All of the single-equation models are estimated without price information, and are based on the 2013 EVS sample. By contrast, all of the demand systems include price information. Depending on the specific approach, estimation is carried out using OLS, nonlinear seemingly unrelated regression using the R package nlsur (Garbuszus, 2017); and using nonparametric kernel methods, as implemented in the R package np (Hayfield and Racine, 2008).

Table 3: Equivalence scale results (* is the elasticity of the interval means).

Approach	AA	AAC	AACC	AACCC	Elasticity
Engel (1895)	2.34	3.46	4.53	5.69	1.09
ELES (Lluch, 1973)	1.70	1.77	1.93	2.17	0.50
FELES (Merz, 1983)	1.54	1.70	2.23	1.39	0.40
QES (Howe et al., 1979)	1.74	1.91	1.74	1.30	0.36
QES (unscaled)	2.00	2.10	2.27	2.54	0.61
AI (Deaton and Muellbauer, 1980)	1.16	1.22	1.24	1.33	0.17
QAI (Banks et al., 1997)	1.47	1.69	1.97	2.11	0.48
Pendakur (1999)	1.22	2.40	2.10	1.16	0.40
Stengos et al. (2006)	1.76	1.62	1.89	2.15	0.48
Dudel (2015)	[1.54;1.74]	[1.66;1.85]	[1.89;2.10]	[1.75;1.90]	0.45*
Modified OECD scale	1.50	1.80	2.10	2.40	0.54

5 Results

5.1 Equivalence scale estimates

Equivalence scale estimates for all methods are presented in Table 3. More specifically, using the household of a single adult as the reference, estimates are shown for childless couples (AA), couples with one child (AAC), couples with two children (AACC), and couples with three children (AACCC). In addition, we calculate the equivalence scale elasticity, which is defined through $S = n^{\alpha}$, where S is the equivalence scale value, n is household size, and α is elasticity (Buhmann et al., 1988). Generally, we can assume that α lies between zero and one, with a value of zero implying that additional household members do not lead to any additional costs, and a value of one implying that there are no economies of scale. For comparisons, the last row of the table includes the modified OECD scale often used by researchers. It has an elasticity of around 0.5.

The single-equation Engel approach yields the highest equivalence scale estimates. Adding a household member always increases the scale by a value of at least one, which is rather implausible. For instance, according to these estimates, a couple needs 2.3 times as much income as a single adult, which implies that the costs of living together are higher per person than the costs of living alone. Not surprisingly, the elasticity implied by these results is around 1.1, and is thus outside of what are generally considered to be reasonable bounds for α. By contrast, the AI demand system, gives the lowest equivalence scale estimates, and an elasticity that is rather low, with a value of 0.2. Again, these results are not very plausible, as they suggest that two people (AA) have roughly the same scale value as four people (AACC).

Meanwhile, the parametric demand systems show some variability, but most generate results that are relatively close to those of the modified OECD scale, as indicated by elasticities of between 0.4 and 0.6. ELES, FELES, QES and QAI yield scale values of between 1.5 and 1.7 for AA (OECD: 1.5), between 1.7 and 1.9 for AAC (OECD: 1.8), and between 1.7 and 2.3 for AACC (OECD: 2.1). For couples with three children (AACCC), the results cover a wider range of between 1.3 and 2.5. These findings may be attributable to the relatively small sample size for this group.

Overall, the semiparametric approaches do not contradict these findings, as the elasticities are between 0.4 and 0.5. Still, the equivalence scale values do not always increase with household size; and for the approach by Pendakur (1999), the estimate for couples with three children is rather implausible. The results from the entirely nonparamteric approach by Dudel (2015) support the idea that the other approaches generate roughly similar results: with the exception of AACCC, QAI, and Stengos et al. (2006), the results are consistent with the results of nonparametric bounds. This is also the case for the modified OECD scale. Overall, it seems that the specific assumptions of the approach that is applied are not very important when the dataset is consistent.

5.2 Inequality and poverty indices

The implications of the different equivalence scale estimates for the measurement of inequality and poverty are shown in Table 4. Each set of equivalence scale values shown in Table 3 was applied to EVS household income data to calculate equivalized income; which was in turn was used to calculate three commonly used indicators: the Gini coefficient, the at-risk-of-poverty rate (ARP), and the interquartile range (IQR).

The modified OECD scale yields values of 0.25 for the Gini coefficient, 14.8% for the ARP, and around 4000 EUR for the IQR.[4] The other equivalence scales lead to indicator values that are both higher and lower. For instance, the equivalence scale resulting from the Engel approach leads to the highest value of the Gini coefficient of 0.28. The ARP value is also relatively high, at close to 18%; while the IQR value is the lowest. By contrast, the AI equivalence scale yields the highest ARP (close to 19%) and IQR estimates (around 6300 EUR), while the value of the Gini coefficient is lower for this scale than for the Engel scale.

As the other approaches lead to less extreme equivalence scale estimates than the Engel and the AI approaches, their results for the inequality and poverty indicators tend

[4]When applied, the weights for children differ. This is considered in the application. The household-specific OECD scale can vary by 0.2 per child depending on the ages of the household members.

Table 4: Measurement of inequality and poverty (* is evaluated at the interval means).

Approach	Gini	ARP	IQR
Engel (1895)	0.28	17.7	3451
ELES (Lluch, 1973)	0.23	14.7	3883
FELES (Merz, 1983)	0.24	14.9	4154
QES (Howe et al., 1979)	0.24	15.2	4149
AI (Deaton and Muellbauer, 1980)	0.27	18.8	6278
QAI (Banks et al., 1997)	0.24	14.9	4161
Pendakur (1999)	0.28	16.2	4990
Stengos et al. (2006)	0.23	14.7	3808
Dudel (2015)*	0.27	17.1	3570
Modified OECD scale	0.25	14.8	3964

Note: ARP in %; IQR in EUR.

to be less spread out. The Gini coefficient ranges from 0.23 to 0.27, the ARP is between 15% and 17%, and the IRQ is around 4000 EUR. Thus, while there is some variability in the results, the equivalence scales with elasticities in the same range produce roughly similar results. There is no clear indication of which class of approaches — parametric, semiparametric, nonparametric — produces the highest estimates of inequality or poverty. To some extent, this is also the case for elasticities. For instance, the FELES and the approach by Pendakur (1999) have rather similar elasticities, but the Gini coefficient is comparatively low when the former approach is applied, and is relatively high when the latter approach is applied. This is because elasticities give only approximations, and the semiparametric scale value for, say, AACCC households is considerably smaller than the FELES estimate.

5.3 Comparison with findings from the literature

For Germany, several studies using expenditure-based approaches and EVS data have been published that cover, some of the methods we use. The AI and QAI demand systems are exceptions, and to the best of our knowledge, no equivalence scale estimates for Germany based on these systems have previously been reported (Missong, 2004). An overview of equivalence scale estimates taken from the literature is provided in Table 5. Note that these estimates are not only based on different methods, but also on different years of EVS data and on different preprocessing steps.

Compared to the scales in other studies using LES-based models (e.g., Lohmann, 2001;

Table 5: Previous equivalence scales based on German expenditure data, Reference household type = A

Source	EVS	Method	A	AA	AAC	AACC	AACCC	Elasticity
Faik (2011)	2003	ELES	1	1.65	1.78	1.92	2.13	0.49
Kohn and Missong (2003)	1988,1993	QES	1	1.73	1.91*	1.75*	2.28	0.50
Lohmann (2001)**	1993	ELES/FELES	1	1.46	1.85	1.95	1.98	0.47
Stryck (1997)	1988	LES/Translating	1	1.43	1.54	1.62	1.64	0.34
Faik (1995)	1983	Engel	1	1.81	2.19	2.45	2.77	0.66
Faik (1995)	1983	Barten	1	1.48	1.73	1.89	1.98	0.45
Faik (1995)	1983	LES/Translating	1	1.34	1.53	1.64	1.72	0.36
Merz and Faik (1995)	1983	LES	1	1.58	1.72	1.85	1.99	0.45
Scheffter (1991)	1983	LES	1	1.35	1.68	1.84	2.03	0.44
Wilke (2006)***	1998	Semiparametric	1	1.54				
Modified OECD scale		Expert-based	1	1.50	1.80	2.10	2.40	0.50

* considering only infants ** considering all commodities *** Nadaraya-Watson Estimator, employed sample

Kohn and Missong, 2003; Missong, 2004), our scales are a bit higher. Our scales also tend to be unstable for couples with more than one child, most likely because of our small sample size. Relative to results based on the translating-approach (e.g., Stryck, 1997; Faik, 1995), our estimates are somewhat lower. Wilke (2006) estimated semiparametric equivalence scales using the EVS of 1998. For AA, he obtained a rate of 1.54 when unemployed persons were excluded. This finding is roughly comparable to our results using the approach by Stengos et al. (2006).

The range of elasticities for the scales found in Table 5 is rather wide, at between 0.34 and 0.66. Our results fall within a similar range. Excluding the two most extreme and rather implausible estimates, our findings are within a slightly narrower range of elasticities, between 0.36 and 0.61. This range is much smaller than those found in other publications. For instance, the often-cited paper by Buhmann et al. (1988) reported a range of between 0.12 and 0.84. These discrepancies suggest that the variability of the equivalence scale estimates found in the literature might be attributable to the use of data from different years and countries, and to differences in preprocessing. By contrast, when the data and the preprocessing are consistent, the estimates tend to be similar, even though some variability remains.

6 Conclusion

In this paper, we compared 10 different approaches for the estimation of equivalence scales, covering parametric, semiparametric, and fully nonparametric methods. Applying these approaches to German expenditure data from the Sample Survey of Income and Expenditure (waves 2003, 2008, 2013), we found a much narrower range of equivalence scale estimates than the previous literature. We attribute our findings primarily to our use of a single database and a consistent approach to the preprocessing of data. Most of our equivalence scale estimates and their elasticities are close to those of the modified OECD scale. Using our equivalence scale estimates to calculate the Gini coefficient, the at-risk-of-poverty ratio, and the interquartile range, we found that most of the equivalence scales lead to rather similar findings.

Although the overall range of our equivalence scale estimates is rather narrow, there are some outliers. It is possible that using a different subset of the EVS for estimation would lead to more plausible results for the methods that we found to be problematic. For instance, if we alter the range to identify outliers and lower the interval to 3.5 times the standard deviation (instead of 2.5), only Engel and ELES and AI vary slightly. Most of the other scales react quite sensitively (see Table 6 in the appendix). This observation highlights a key problem associated with the subset selection commonly employed in household economics: namely, that data editing and restricting analyses to specific subsets of the data, or data preprocessing, can greatly affect the results.

While we covered several very different approaches, our conclusions are restricted to a limited set of methods only; and many methods have been proposed in the literature that we were not able to include here. For instance, in single-equation models, the share of expenditures on food may be replaced with other indicators of household welfare, most notably expenditures on goods exclusively consumed by adults. This would allow us to distinguish between the equivalence scales of households with and without children (Rothbarth, 1943). Moreover, specifications other than the Working-Leser specification have been proposed, some of which make the resulting equivalence scales income dependent (Lancaster and Ray, 1998).

For researchers applying single exact equivalence scales, using the modified OECD scale can be seen as a reasonable choice if an income-independent scale is desired, at least for Germany. While our estimates vary from those of the OECD scale, the differences appear to be small when measuring inequality and poverty.

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A Appendix

Table 6: Equivalence scales with outlier $\neq [3.5 \times SD]$ (n = 32,044; * is evaluated at the interval means).

Approach	AA	AAC	AACC	AACCC	Elasticity
Engel (1895)	2.28	3.41	4.31	5.54	1.07
ELES (Lluch, 1973)	1.70	1.76	1.92	2.11	0.49
FELES (Merz, 1983)	1.50	1.66	1.54	1.43	0.31
QES (Howe et al., 1979)	1.73	1.66	1.44	1.54	0.33
QES (unscaled)	1.80	1.91	2.09	2.50	0.57
AI (Deaton and Muellbauer, 1980)	1.18	1.22	1.29	1.36	0.19
QAI (Banks et al., 1997)	1.53	1.66	2.02	2.25	0.50
Pendakur (1999)	1.43	0.99	1.9	1.03	0.21
Stengos et al. (2006)	1.78	1.40	1.7	2.96	0.56
Dudel (2015)	[1.54;1.78]	[1.64;1.84]	[1.90;2.13]	[1.73;1.91]	0.45*
OECD	1.50	1.80	2.10	2.40	0.54