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Ansgar Belke and Matthias Göcke[1]

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Abstract

The interest rate is generally considered as a monetary policy tool and, at the same time, via Tobin's q, as an important driver of macroeconomic investment. As an innovation, this paper derives the exact shape of the "hysteretic" impact of changes in the interest rate on macroeconomic investment under the scenarios of certainty and uncertainty. We capture the direct interest rate-hysteresis effects on investment and the capital stock and, explicitly, stochastic changes of the interest rate-investment hysteresis relationship. Starting with hysteresis effects on a microeconomic level of a single firm, we apply an explicit aggregation procedure to derive the interest rate-hysteresis effects on a macroeconomic level. Based on our simple model we are quite skeptical regarding the efficacy of the central bank in providing incentives for macroeconomic investment in times of low or even zero interest rates and high uncertainty. Only if the central bank implements monetary policy strategies such as "forward guidance" and is able to credibly commit to low interest rates also for the foreseeable future, our quite strong verdict may be of less relevance.

JEL Classification: C61, E22, E44

Keywords: Forward guidance; interest rate; investment; Mayergoyz-Preisach model; monetary policy; path dependence; non-ideal relay; sunk-cost hysteresis; uncertainty; zero lower bound

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1. Introduction

Relations between economic variables are often characterized by a scenario where initial conditions and the past realizations of economic variables matter. I.e. past (transient) exogenous disturbances and past states of the economic system do have an influence on the current economic relations. Typical examples are the dynamics of (un)employment in business cycles and the dynamics of the nexus of exchange rates and exports. However, the focus is to an increasing extent, from the perspective of monetary policy, also on the dynamics of the relation between the interest rate (which is to a certain extent influenced by the central bank itself) and investment. Since the standard characteristics of hysteresis apply – i.e. permanent effects of a temporary stimulus, resulting in path-dependent multiple equilibria – these economic phenomena are termed “hysteresis” (Göcke, 2002, Belke et al. 2014).

At the same time, uncertainty has gained much attention as one of the main drivers of the depth and duration of the Great Recession (Bloom et al., 2013, Caggiano et al. 2017). In this context, there is a growing interest in the impacts of (policy) uncertainty on monetary policy and on hysteretic systems such as capital and labor markets, especially in the context of sustained lower growth (“secular stagnation”). Lawrence Summers, to cite a prominent example, has stated repeatedly that the phenomenon of “secular stagnation” cannot be fully explained without taking recourse to hysteresis (Belke, 2018, Buiter et al. 2015, Summers, 2014, 2015). Hence, it appears to be highly rewarding to investigate the mechanics of interest rate hysteresis in macroeconomic investment under uncertainty and, on a more cursory level, the *implications for monetary policy* more deeply.

Analogous to magnetism, the pattern of hysteresis depends on the scope: based on sunk-adjustment costs (e.g. entry costs of starting investment activity) *micro*economic behavior (e.g. of *single firms* as investors) may show a discontinuous switching-pattern (being active as an investor or not) as described by a non-ideal relay, analogous to the magnetism of a single iron crystal. Correspondingly, one may conjecture that the *macro*economic dynamics of *aggregate* economic variables (e.g. the investment activity of a *whole country,* based on an aggregation over firms with heterogeneous cost structures) shows a pattern similar to the well-known hysteresis-loop of an entire piece of iron. The aggregate macroeconomic loop would then be characterized by a smooth/continuous transition between different “branches” of the loop, occurring with changes in the direction of, for instance, the interest rate movement.

However, the technical problem we are faced with is that hysteresis in economics is up to now usually based on a representation of a system with only a single input variable, which has an enduring effect on an economic outcome (i.e. the output variable). This input variable typically stands for a price or earnings variable (e.g. the exchange rate, affecting unit earnings in foreign trade, or wages as the price or cost of labour) in the context of employment hysteresis. However, in general there is more than one factor influencing economic decision problems. We call this constellation the “multiple input variable scenario”.

Since hysteresis problems are about investment decisions in the broadest sense, the interest rate is particularly important, since the profitability depends on interest rates as the main determinant of the cost of capital. As a stylized fact, the central bank is able to influence investment activity via the asset price channel and the impact of its interest rate setting on Tobin’s q (Hayashi, 1982). In other words, the central bank’s policy rate can be considered as a driver of the market interest rate i which is in the focus of our modelling efforts in this paper.

The “multiple input variable scenario” (i.e. “vector hysteresis”) in economics has been addressed by Göcke (2018), where it is outlined how the influence of several original input variables (e.g. the revenue level and interest rate) is captured by the resulting variations of the present value of an investment. However, following Belke/Göcke (2009), we want to focus only on hysteretic nature of the relation between investments and the interest rate in the present paper. Starting with a microeconomic model, we show the path-dependent nature of investment decisions, resulting in a difference between the (low) interest rate that triggers an investment and the (high) interest rates that triggers a disinvestment of a single firm. The divergence between these both interest triggers may be due to sunk cost, e.g. if the investment is firm specific and cannot be sold at the full purchasing price.

In a situation with uncertainty, e.g. related to stochastic future revenues and/or to future interest rate changes, the hysteresis property is even amplified due to option value effects. While Belke/Göcke (2009) calculate simple ‘symmetric’ stochastic revenue changes only for the microeconomic firm level, we *augment* the established model in *three aspects*. First, we explicitly include *stochastic interest rate changes*. Second, to make the model able to illustrate the present monetary policy stance in the Euro area at the zero lower bound (ZLB), we integrate *‘asymmetric’ future changes of the interest rate* that allows a representation of a situation with very low interest rates, where a future increase is possible (and more or less probable) but no further decrease. Third, we come up with a new application of an *explicit* *aggregation procedure* to allow the derivation of macroeconomic hysteresis dynamics between aggregate investments and interest rate changes.

In order to derive the exact shape of the impact of changes in the interest rate (to a certain extent driven by the central bank) on macroeconomic investment under the scenarios of both certainty and uncertainty, we proceed as follows. In section 2, we deal with the representation of sunk cost hysteresis by means of non-ideal relays. For this purpose, we differentiate between scenarios of (1) sunk cost hysteresis and interest changes in a situation with no uncertainty, (2) non-ideal relay in a situation with stochastic revenue changes, (3) non-ideal relay in a situation with stochastic interest rate changes and (4) non-ideal relay related to interest rates in a stochastic situation. In Section 3, we aggregate our micro results to the macroeconomic level and formally derive the main pattern of macroeconomic, interest rate-driven investment hysteresis. Section 4 finally concludes and draws some implications for monetary policy and its transmission to the real sector (investment), for instance in the Euro area.

2. Sunk cost hysteresis and non-ideal relays

2.1 Sunk cost hysteresis and interest changes in a scenario with no uncertainty

In order to illustrate the hysteresis effects on a microeconomic level, we will apply a simplistic microeconomic model that shows a couple of similarities to the model presented by Belke/Göcke (2009). A price-taking firm j decides in period t whether or not to invest into one unit of capital $K_j (= 1)$. Additionally, the firm has to pay the sunk investment costs H_j (≥ 0) if it invests and starts production. The value created by H_j is completely firm specific and can not be regained if the firm is disinvesting. If production is shut down, selling the unspecific capital stock K_j at price 1 is possible. The specific part H_j decays immediately as soon as the firm does not produce and sell. Thus, H_j represents sunk adjustment costs. Using capital, the firm produces and sells the production immediately resulting in a revenue $e_{j,t}$. Since one additional unit of capital is applied, $e_{j,t}$ is the (marginal gross) rate of return. Two different components of costs have to be paid. Based on using unspecific capital as an input factor, the interest rate i_t (influenced by the central bank) has to be paid on the firm's capital stock K_j as an opportunity cost. Additionally, if the firm has not produced in the preceding period, it has to pay the starting costs H_j (≥ 0). On the other hand, if it has been active in the preceding period and is just continuing production only the interest costs on unspecific K_j are relevant.

The net rate of profit (disregarding the adjustment costs) in period t is:

$$(1) \qquad R_{j,t} = e_{j,t} - i_t \cdot K_j \; = \; e_{j,t} - i_t$$

As a simple example we assume the firm is expecting with certainty for the next period (t+1) a single “once and forever” change in the interest rate (imposed, for instance, by announcements of the central bank) by ρ, which remains constant for the whole *infinite future*: $i_{t+\tau} = i_t + \rho$ (for all $\tau > 0$). The future gross rate of return is (with certainty) expected constant as well: $e_{j,t+\tau} = e_{j,t}$. Thus, the future net rate of return is:

$$(2) \qquad R_{j,t+\tau} = e_{j,t} - i_{t+\tau} = e_{j,t} - (i_t + \rho) \qquad (\text{with } \tau > 0)$$

Under the assumed/expected interest rate dynamics, the present value of future revenues as an annuity (with payments at the end of the periods) is in the case of an ongoing activity:

$$(3) \qquad V_{j,t} \equiv \frac{e_{j,t} + \dfrac{e_{j,t}}{i_t + \rho}}{1 + i_t} - K_j \; = \; \frac{(1 + i_t + \rho) \cdot e_{j,t}}{(1 + i_t) \cdot (i_t + \rho)} - 1$$

Under certainty the present value of revenues has to cover (at least) the value of the capital stock $K_j = 1$ plus the sunk entry costs ($V_{j,t} > K_j + H_j$) to make an entry a profitable investment. Solving ($V_{j,t} = 1 + H_j$) leads to the firm’s investment/entry trigger rate of return α_j under certainty:

$$(4) \qquad \alpha_j = \frac{(1 + i_t) \cdot (i_t + \rho) \cdot (1 + H_j)}{1 + i_t + \rho} \qquad\qquad \text{entry if } e_{j,t} > \alpha_j$$

Therefore, the (gross) rate of return has to cover the interest cost on both, the unspecific capital K_j plus the sunk investment costs H_j.

A firm that was active in the preceding period will leave the market and sell the unspecific capital K_j, if the revenue is too low. Thus, an exit of the firm is optimal if ($V_t < 1$), and the exit/disinvestment trigger rate of return β_j is:

$$(5) \qquad \beta_j = \frac{(1 + i_t) \cdot (i_t + \rho)}{1 + i_t + \rho} \qquad\qquad \text{exit if } e_{j,t} < \beta_j$$

In the even simpler case of an expectation of unchanged and constant future interest rates ($\rho = 0$, i.e. the central bank credibly commits itself to a constant interest rate path in the future), the entry and exit triggers of the gross rate of return are:

(6) for $\rho = 0$: $\alpha_{j,\rho=0} = i_t \cdot (1 + H_j)$ entry if $e_{j,t} > \alpha_j$

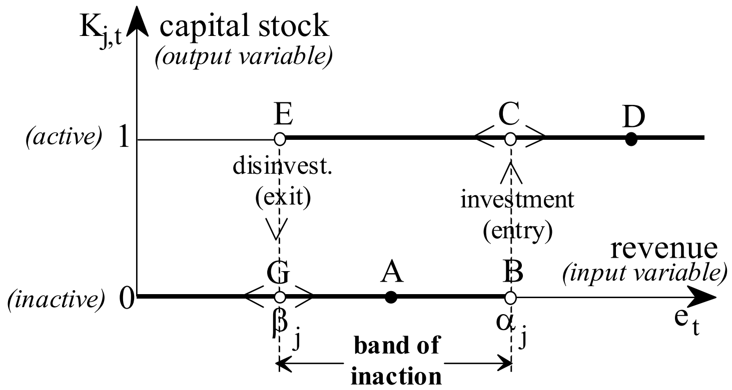
$\beta_{j,\rho=0} = i_t$ exit if $e_{j,t} < \beta_j$

The results show, that an entry requires interest costs on both, unspecific capital (normalized to $K_j = 1$) and sunk investment costs ($H_j \geq 0$), based on the future interest rate ($i_t + \rho$) to be covered by the revenue, i.e. by the gross rate of return $e_{j,t}$, to make an investment profitable. On the other hand an exit will only occur, if the rate of return $e_{j,t}$ no longer covers the interest opportunity costs just on the unspecific part of the capital K_j. Thus, a disinvestment occurs if the rate of return falls below the future interest rate.

Summarising, if interest rate expectation is fixed, an (unexpected) change in the (current and future) rate of return on capital $e_{j,t}$ results in an investment/disinvestment pattern of the firm j which is described by a so called 'non-ideal relay':[1]

(7) $$K_{j,t} = \begin{cases} 1 & \text{if } (K_{j,t-1} = 0 \wedge e_{j,t} > \alpha_j) \\ 1 & \text{if } (K_{j,t-1} = 1 \wedge e_{j,t} \geq \beta_j) \\ 0 & \text{if } (K_{j,t-1} = 0 \wedge e_{j,t} \leq \alpha_j) \\ 0 & \text{if } (K_{j,t-1} = 1 \wedge e_{j,t} < \beta_j) \end{cases} \quad \text{with } \alpha_j \geq \beta_j$$

Fig. 1: Investments according to a 'non-ideal relay' related to the rate of return as the input variable



[1] For a general description of relay-hysteresis see Krasnosel'skii/Pokrovskii (1989, p. 263, and p. 271) and Brokate/Sprekels (1996, pp. 23 f.)

A non-ideal relay describes a path-dependent multiple-equilibria characteristic. Starting in an inactivity situation at point A (Fig. 1) a revenue increase exceeding the trigger α_j induces an investment ("entry"), i.e. a "jump" from the (K_j=0)-inactivity-line to the (K_j=1)-activity-line (point C). A later revenue decrease is resulting in disinvestment ("exit", point E), only if the rate of return falls below the exit trigger β_j. A switch between the two equilibrium-branches takes place when the triggers are passed – otherwise the activity status remains the same. Therefore, the area GB (or CE) can be described as a *'band of inaction'* or 'hysteresis-band' (Baldwin, 1989, pp. 7 f.; Baldwin/Lyons, 1989, p. 11.). Dependent on the past, two different equilibria are possible: The current level of the input variable (revenue) does not unambiguously determine the current state of the output/dependent variable (firm's activity). If a *temporary* change of the input variable results in a switch between these equilibria, a *permanent* effect on the output variable remains (called '*remanence*'). This after-effect is the constituting feature of hysteresis.

Up to now, the level of the rate of return $e_{j,t}$ was implicitly assumed as the single input variable of the system, and the entry trigger condition ($V_{j,t}=1+H_j$) was solved for $e_{j,t}$ in order to derive revenue triggers. However, if alternatively the interest rate is assumed to be the single input variable, while the rate of return $e_{j,t}$ is ceteris paribus expected as constant ($e_{j,t}=e_j$), the entry condition can be solved for the interest rate, and an entry trigger interest rate a_j can be calculated.[2] We assume the simplest case of $\rho=0$ for illustration. The entry/investment trigger interest rate is:

$$(8) \qquad \text{for } \rho=0: \quad a_{j,\rho=0} = \frac{e_j}{1+H_j} \qquad\qquad \text{entry if } i_t < a_j$$

A low interest rate results in low capital costs for K_j and H_j and in a high present value of future revenues making an investment profitable. A similar calculation determines the disinvestment interest trigger rate b_j for a scenario with a high interest rate:

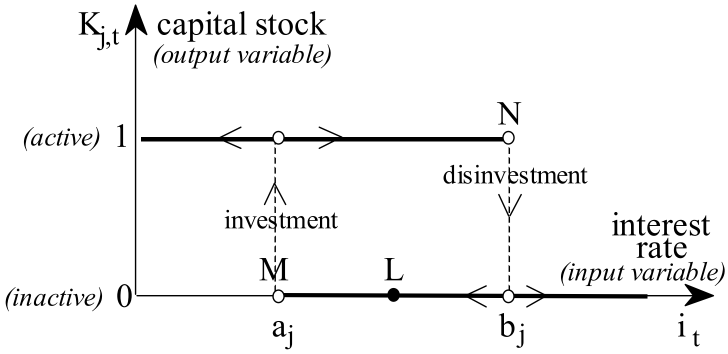
$$(9) \qquad \text{for } \rho=0: \quad b_j = e_j \qquad\qquad \text{exit if } i_t > b_j$$

[2] See Göcke (2018) for a discussion of the interest rate related microeconomic hysteresis loop. In that paper a multiple inputs situation, with variations in the revenue as well as in the interest rates, is addressed. However, his "vector-hysteresis" with multiple inputs is shown to be simplified in economic problems, if simultaneous changes of the interest rate and the revenues are reduced to changes of a one-dimensional "signal" presented by the present value of the investment.

Thus, for a constant level of the rate of return $e_{j,t}=e_j$ an (unexpected) change of the interest rate i_t results again in a 'non-ideal relay' pattern, this time with respect to the interest rate as the input variable as depicted in Fig. 2 (see Göcke, 2018, p. 6, for a similar representation):

$$(10) \qquad K_{j,t} = \begin{cases} 1 & \text{if } (K_{j,t-1} = 0 \wedge i_t < a_j) \\ 1 & \text{if } (K_{j,t-1} = 1 \wedge i_t \leq b_j) \\ 0 & \text{if } (K_{j,t-1} = 0 \wedge i_t \geq a_j) \\ 0 & \text{if } (K_{j,t-1} = 1 \wedge i_t > b_j) \end{cases} \qquad \text{with } a_j \leq b_j$$

Fig. 2: 'Non-ideal relay' related to the interest rate as the input variable



Again, a difference between an investment and disinvestment trigger level of the input variable emerges, which results in a range of path-dependent multiple equilibria (i.e. an interest rate related "band-of-inaction" between a_j and b_j). Starting in point L, a decreasing interest rate, makes an investment profitable (at point M) by reducing interest costs on both types of capital (K_j and H_j). Future interest rate increases will result in disinvestment (point N with exit trigger rate b_j), if the opportunity costs only on the unspecific K_j are no longer covered by revenues.

2.2 Non-ideal relay in a scenario with stochastic revenue changes

In a situation with uncertainty, e.g. due to expected stochastic changes in future revenues, a real option approach applies (Pindyck, 1988, 1991; Dixit, 1989; Bentolila/Bertola, 1990; Dixit/Pindyck, 1994, Belke/Göcke, 1999, 2009). We again apply the simple case of no expected change in the interest rate (ρ=0) for illustration of the option effects related to revenue uncertainty. A firm which is currently inactive has to decide whether to invest now or not, including the option to invest later. The option to decide on the investment in the future limits the risk by a "wait-and-see" strategy. By staying passive, a firm can avoid future losses if the stochastic future rate of return will be unfavourable. An instantaneous investment eliminates this "wait-and-see"-option to enter later if the future revenue will be favourable. Thus, the option value of waiting has to be covered in addition to the sunk costs in order to trigger an immediate investment.

The option value effects are demonstrated based on a very simplistic example: assume a single non-recurring stochastic change of the rate of return, which can be either positive ($+\varepsilon_u$) or negative ($-\varepsilon_d$) (and $\varepsilon_{u,d} \geq 0$). The probability of a negative change ($-\varepsilon_d$) is P, and $(1-P)$ for a positive revenue change ($+\varepsilon_u$). From period t+1 on, the firm will decide under certainty again. Instead of deciding to invest now or never, the option to wait and to decide on the investment later has to be taken into account in a scenario with uncertainty. If the future revenue level turns out to be favourable ($+\varepsilon_u$) the firm can still invest in the next period. However, by staying passive, potential future losses can be avoided if the revenue change will be negative ($-\varepsilon_d$). To wait and stay passive implies zero profits in the current period t. Conditional on a ($+\varepsilon_u$)-realisation, the firm will use its option to invest in t+1. This causes discounted sunk investment costs and an annuity of ($e_{j,t}+\varepsilon_u-i_t$). In case of a ($-\varepsilon_d$)-realisation the firm will remain passive. The expected present value of the wait-and-see strategy is $E(W_{j,t}^{entry})$ and has to be compared with the expected present value $[E(V_{j,t})-H_j]$ of an immediate entry (without a re-exit):

$$(11) \qquad E(W_{j,t}^{entry}) = \frac{1-P}{1+i_t} \cdot \left(\frac{e_{j,t}+\varepsilon_u-i_t}{i_t} - H_j \right)$$

$$(12) \qquad E(V_{j,t})-H_j = P \cdot \left(\frac{e_{j,t} + \frac{e_{j,t}-\varepsilon_d}{i_t}}{1+i_t} \right) + (1-P) \cdot \left(\frac{e_{j,t} + \frac{e_{j,t}+\varepsilon_u}{i_t}}{1+i_t} \right) - 1 - H_j$$

In the case of uncertainty, the investment revenue trigger α_j can be calculated for a situation of indifference between immediate entry and wait-and-see, i.e. if $E(W_{j,t}^{entry}) = E(V_{j,t})-H_j$:

$$(13) \qquad \alpha_j = i_t \cdot (1+H_j) + \frac{P \cdot \varepsilon_d}{i_t+P} \qquad \text{in period t: entry if } e_{j,t} > \alpha_j$$

A currently active firm, deciding to leave the market now or to stay active, with an option to exit later if an unfavourable ($-\varepsilon_d$) price change will occur, has an analogous decision problem. Currently remaining active and waiting for a period results in a current profit of ($e_{j,t}-i_t$). Conditional on a ($-\varepsilon_d$)-realisation, the firm will use its option to exit in t+1. For a ($+\varepsilon_u$)-

realisation the firm will continue activity with a future annuity of $(e_{j,t}+\varepsilon_u-i_t)$. The expected present value of the wait-and-see strategy $E(W_{j,t}^{exit})$ is:

$$(14) \qquad E(W_{j,t}^{exit})_{j,t} = P\cdot\left(\frac{e_{j,t}-i_t}{1+i_t}\right)+(1-P)\cdot\left(\frac{e_{j,t}+\dfrac{e_{j,t}+\varepsilon_u}{i_t}}{1+i_t}-1\right)$$

The present value of waiting $E(W_{j,t}^{exit})$ has to be compared with the expected present value of an immediate exit (without a re-entry), which is zero. The exit/disinvestment-trigger revenue level β_j in the case of revenue uncertainty can be calculated for a situation of indifference between wait-and-see and immediate exit in t, i.e $E(W_{j,t}^{exit})=0$:

$$(15) \qquad \beta_j = i_t - \frac{(1-P)\cdot\varepsilon_u}{i_t+(1-P)} \qquad \text{in period t: exit if } e_{j,t}<\beta_j$$

Thus, the investment trigger rate of return α_j is under uncertainty augmented by the positive term $[+(P\cdot\varepsilon_d)/(i_t+P)]$, and the option value effect on the disinvestment trigger revenue β_j is negative: $-(1-P)\cdot\varepsilon_u/[i_t+(1-P)]$. Thus, the option value effects due to (revenue) uncertainty result in a widening of the ('band-of-inaction')-range between both triggers β_j and α_j. However, considering the typical pattern of path-dependent multiple equilibria, the non-ideal-relay dynamics of microeconomic hysteresis do not have changed.

2.3 Non-ideal relay in a scenario with stochastic interest rate changes

A qualitatively similar widening effect on the 'band of inaction' will result, if uncertainty is not based on stochastic revenue changes, but *on stochastic future changes of the interest rate* (for instance, induced by – from a firm's perspective stochastic – future central bank policy changes). Moreover, the same consequence of widening the ('band-of-inaction')-range results due to uncertainty induced option value effects for the distance between the entry and exit trigger interest rate a_j and b_j.

The option value effects are again demonstrated by a simplistic example: now a single non-recurring stochastic change of the interest rate, which can be either positive $(+\rho_u)$ or negative $(-\rho_d)$ (and $\rho_{u,d}\geq 0$). The probability of a positive interest change $(+\rho_u)$ is P, and $(1-P)$ for a negative interest rate change $(-\rho_d)$. From period t+1 on, the situation is assumed to be unchanged/constant again. The option to wait and to decide on the investment later has again to be taken into account in this scenario. If the future interest change is negative ($-\rho_d$), and thus favourable for investors, the firm can still invest in the next period, and by staying passive, potential future losses can be avoided if the positive interests change ($+\rho_u$) is related to higher increasing interest cost. Regarding eq. (1) the current net profit is $R_{j,t}=e_{j,t}-i_t$. The future interest rate, relevant for the interest costs and for future discounting, is (with probability P) $i_{t+\tau}=i_u\equiv i_t+\rho_u$ and with probability (1–P): $i_{t+\tau}\equiv i_d=i_t-\rho_d$. The future profit follows from eq. (2), if $i_t+\rho$ is substituted by either $i_t+\rho_u$ or $i_t-\rho_d$. Using eq. (3), the resulting present values of future revenues for both possible cases can be calculated by the same substitution:

$$(16)\qquad V_{t,u}\equiv\frac{(1+i_t+\rho_u)\cdot e_{j,t}}{(1+i_t)\cdot(i_t+\rho_u)}-1 \quad\text{and}\quad V_{t,d}\equiv\frac{(1+i_t-\rho_d)\cdot e_{j,t}}{(1+i_t)\cdot(i_t-\rho_d)}-1$$

A currently inactive firm decides to invest now or not, including the option to invest in the next period, if the interest costs are decreasing – i.e. conditional on a ($-\rho_d$)-realisation of the stochastic interest rate, which has a probability of $(1-P)$. The expected present value of the wait-and-see strategy is $E(W_{j,t}^{entry})$ and is compared with the expected present value $E(V_{j,t})$ of an immediate entry $[E(V_{j,t})-H_j]$:

$$(17)\qquad E(W_{j,t}^{entry})=\frac{1-P}{1+i_t}\cdot\left(\frac{e_{j,t}}{i_t-\rho_d}-H_j\right)$$

$$(18)\qquad E(V_{j,t})-H_j=P\cdot V_{t,u}+(1-P)\cdot V_{t,d}$$

The investment revenue trigger α_j results for indifference between both alternatives as:

$$(19)\qquad \alpha_j=\frac{(i_t+\rho_u)\cdot(i_t+P)\cdot(1+H_j)}{i_t+\rho_u+P} \qquad \text{in period t: entry if } e_{j,t}>\alpha_j$$

$$=i_t\cdot(1+H_j)+\frac{\rho_u\cdot P\cdot(1+H_j)}{i_t+\rho_u+P}$$

The result for the investment trigger rate of return shows that for the decision whether or not to invest now (!), instead of waiting for a while, only the potential increase of the interest rate (caused, for example, by a hike of the central bank’s policy rate) is relevant. In this case, an immediate investment later will later turn out to be “wrong” due to increasing costs of the capital. This potential risk can be avoided just by waiting with an option to decide on the investment later, when the future interest rate (i.e. the monetary policy stance) is known. On the other hand, with regard to the investment decision, the chance of a decreasing interest rate is not relevant for an immediate entry, since for this case there is no risk of having conducted the "wrong" investment. Thus, for the investment decision, the option value of waiting is only based on avoiding the stochastic risk of higher future interest rate (i.e. more contractionary monetary policy *in the future*).

A currently active firm can disinvest now or stay active, with the option to leave later, if an unfavourable interest rate increase ($+\rho_u$) will happen. Currently remaining active and to wait up to the next period results in a current profit of ($e_{j,t}-i_t$). Conditional on a ($+\rho_u$)-realisation, the firm will use its disinvestment option in t+1. For a ($-\rho_d$)-realisation the firm will continue activity with a future annuity of ($e_{j,t}-i_t+\rho_d$) and the corresponding present value $V_{t,d}$. The expected present value of the wait-and-see strategy $E(W_{j,t}^{exit})$ is:

$$\text{(20)} \qquad E(W_{j,t}^{exit}) = P \cdot \left(\frac{e_{j,t}-i_t}{1+i_t} \right) + (1-P) \cdot V_{t,d}$$

An immediate disinvestment results in zero profits. Hence, indifference is given with $E(W_{j,t}^{exit})$ = 0, determining the exit trigger:

$$\text{(21)} \qquad \beta_j = \frac{(i_t-\rho_d)\cdot(i_t+1-P)}{i_t-\rho_d+1-P} = i_t - \frac{\rho_d \cdot (1-P)}{i_t-\rho_d+1-P} \qquad \text{in period t: exit if } e_{j,t}<\beta_j$$

The results for the immediate disinvestment versus continuing activity for a while with an option to exit later, is only related to the potential decrease of the interest rate (i.e. induced by a lower monetary policy rate). An immediate disinvestment would turn out as the "wrong" decision, if the future interest cost decreases, while an immediate exit would turn out to be right in the case of an increasing interest rate. Waiting prevents the risk of a "wrong" decision in the case of reduced interest costs. Hence, only the risk of decreasing future interest rates (i.e. a more expansionary future monetary policy stance) is relevant for an immediate disinvestment. In this case, the option value of waiting is merely based on avoiding the stochastic risk of a lower future interest rate.

In a situation with no stochastic change ($+\rho_u=-\rho_u=0$), the results resemble the triggers stated in eq. (6) where the special case with no expected future interest rate change ($\rho=0$) is assumed: $\alpha_{j,\rho=0}=i_t\cdot(1+H_j)$ and $\beta_{j,\rho=0}=i_t$. Again, as in the case of stochastic revenue changes, option value effects widen the hysteretic "band of inaction". The entry trigger α_j is increased by an $[(+\rho_u)\cdot P]$-based term, describing the danger of an unfavourable interest rate increase which could be avoided by waiting. The exit trigger β_j is reduced related to a $[(-\rho_d)\cdot(1-P)]$-term, since there may be a favourable potential future interest rate decrease.

The results have clear implications for monetary policy if we consider the *consequences of an interest rate close to zero* (the so-called "zero lower bound"), as experienced for years in the US and other industrialised countries in the wake of the global financial crisis and still in the Euro area, *as an investment incentive*. At first glance, an interest rate level near the "zero lower bound" should completely avoid the interest costs on an investment, and thus, result in massive investment activities, if the rate of return on investments are only slightly positive. However, even if the current interest rate is $i_t=0$, there is still a risk of future interest rate increases (i.e. policy rate hikes by the central bank), which in combination with the sunk investment costs H_j results in an investment trigger of the rate of return that is still pretty far in the positive range, since the investment still not only has to cover the interest costs (which are zero in this specific situation), but additionally the option value of waiting. This option value is driven by the potential size ρ_u of the interest rate increase, and by its probability P:

$$\text{(22)} \qquad \text{for } i_t=0: \quad \alpha_{j,\, i=0} = \frac{\rho_u \cdot P \cdot (1+H_j)}{\rho_u + P} > \mathbf{0}$$

This result shows that in a stochastic environment a monetary policy stance based on low current interest rates (as currently in the Euro area) may be not very effective in stimulating investments, if the future includes the risk of subsequent interest rate increases (i.e. policy rate hikes), creating an option value of waiting with investments.

The exit trigger in an uncertain situation is related to an option value based on potential interest rate decreases $(-\rho_d<0)$, occurring with probability $(1-P)$, mathematically resulting in an even negative rate of return triggering disinvestments:

$$\text{(23)} \qquad \text{for } i_t=0: \quad \beta_j = \frac{-\rho_d \cdot (1-P)}{-\rho_d + 1 - P} \leq \mathbf{0}$$

However, if in a situation with zero interest rates, a further interest rate decrease is not feasible [i.e. $\rho_d \to 0$], or not very probable $[(1-P)\to 0]$, the disinvestment trigger of the rate of return on investment will then converge to zero as well.

2.4 Non-ideal relay related to interest rates in a stochastic scenario

In the previous two subsections, we solved the entry/exit triggers of the non-ideal relay for the rate of return $e_{j,t}$ as the relevant input variable of the hysteresis-system, triggering investments and disinvestments, respectively. Of course, the underlying triggering conditions could be solved for the interest rates as the alternative input variable of the system as well, resulting in a kind of “mirrored” non-ideal relay pattern, as depicted in Fig. 2.

For the case of revenue uncertainty indifference between immediate entry and wait-and-see, $E(W_{j,t}^{entry}) = E(V_{j,t}) - H_j$, results in:

$$(13') \quad e_{j,t} = i_t \cdot (1+H_j) + \frac{P \cdot \varepsilon_d}{i_t + P} \quad \Leftrightarrow \quad i_t = \frac{1}{1+H_j} \cdot \left(e_{j,t} - \frac{P \cdot \varepsilon_d}{i_t + P} \right)$$

In the case of interest rate uncertainty the entry trigger condition $E(W_{j,t}^{entry}) = E(V_{j,t}) - H_j$, is:

$$(19') \quad e_{j,t} = i_t \cdot (1+H_j) + \frac{\rho_u \cdot P \cdot (1+H_j)}{i_t + \rho_u + P} \quad \Leftrightarrow \quad i_t = \frac{e_{j,t}}{1+H_j} - \frac{\rho_u \cdot P}{i_t + \rho_u + P}$$

Explicitly solving these conditions for the current interest rate i_t to calculate the interest rate entry trigger a_j will lead to a confusing result including some root expressions. However, the direction of the option value effects on the level of the interest rate triggering an investment is obvious for both different cases of stochastic effects. Eqs. (13’) and (19’) make clear that there is a negative effect of the option value of potentially decreasing future rate of return and of a potentially increasing future interest rate on the current interest rate that triggers an immediate investment. In Fig. 2 this would be depicted as a shift of the investment trigger a_j to the left.

Analogously, solving the indifference condition between an immediate exit and wait-and-see, $E(W_{j,t}^{exit}) = 0$:

$$(15') \quad i_t = e_{j,t} + \frac{(1-P) \cdot \varepsilon_u}{i_t + (1-P)}$$

$$(21') \quad i_t = e_{j,t} + \frac{\rho_d \cdot (1-P)}{i_t - \rho_d + 1 - P}$$

For the disinvestment trigger we see a positive option value effect on the level of the exit trigger interest rate in both cases of different uncertainty sources, the risk of rising future rates of return and the risk of future interest rate decreases, creating an option value of waiting with the disinvestment decision. The interest rate b_j that triggers an immediate exit is shifted to the right in Fig. 2 in both cases. Hereby, the option value effects lead to a widening of the “band of inaction" related to the interest rate triggers, as it is similarly the case for the band of inaction related to the revenue based triggers.

3. Aggregation and macroeconomic investment hysteresis

So far, we depicted explicitly only the microeconomic effects of hysteresis, resulting in a non-ideal relay dynamics in both perspectives, related to the revenues as well as to interest rates triggering an immediate investment or disinvestment. For macroeconomic dynamics the Mayergoyz (1986)-Preisach (1935)-procedure is typically applied – describing an explicit aggregation procedure for a multitude of heterogeneous non-ideal relay agents ($j = 1, ..., n$; $n \gg 0$), having different entry/exit triggers due to differences in the firms' cost or revenue structures.[3] The standard procedure is based on the microeconomic relay-type as depicted in Fig. 1 for revenue variations, where the entry trigger has a higher level compared to the exit trigger ($\alpha_j \geq \beta_j$) for all heterogeneous firms. Since this relation is "mirrored" if the triggers are stated in terms of interest rates as shown in Fig. 2, with an investment/entry trigger below the exit/disinvestment trigger ($a_j \leq b_j$), the standard Preisach aggregation procedure needs some modifications. We present this modified version (where the axes of coordinates of the exit and the entry trigger were changed in comparison to the standard case based on revenue triggers) below.

Every potentially active firm j is characterized by a twin-set of entry/exit triggers (a_j/b_j). In an a_j/b_j-diagram (see Fig. 3), the firms are represented by points in a triangle area above the 45°-line, if the exit trigger interest rate b_j is depicted on the ordinate axis (since $a_j \leq b_j$) The aggregation procedure can be performed without any serious restriction of the heterogeneity of the firms' distribution over the triangle area (i.e. of the sunk cost H_j and revenue $e_{t,j}$ structure of the firms). Points on the 45°-line describe non-hysteretic firms ($H_j = 0 \Rightarrow a_j = b_j$). A non-ideal relay characterizes firms with a position above the 45°-($a = b$)-line. For reasons of simplicity we assume the firm specific revenues ceteris paribus during the aggregation procedure as constant ($e_{j,t} = e_j$), thus the points representing a firm's capital stock activity pattern remain in their position.

3 This procedure was introduced to economics by Amable et al. (1991) and Cross (1993). See e.g. Göcke (1994), Piscitelli et al. (2000), Belke/Göcke (2005), and Mota/Vasconcelos (2012) for applications of the Preisach-Mayergoyz Procedure in foreign trade and in labour market economics.

Just to avoid a long description of the past development, we assume an initial situation with a very high interest rate ($i_0 >> 0$) as a starting point, implying that no firm is initially active. Now, a decreasing interest rate (i.e. as a consequence of an expansionary monetary policy) makes investments affordable and results in investments/entries by firms with the highest rates of return e_j – i.e. the highest investment interest rate triggers a_j. The aggregate capital stock increases (and if this capital stock is used for production, aggregate output supply of the entire economy), as traced in Fig. 3 (a). This is graphically shown by a growing space of the hatched triangle S_t^+ representing the active firms which have invested and increased their capital stock (while S_t^- represents the inactive firms). For a decreasing current interest rate (i.e. an expansionary monetary policy), the S_t^+-expansion is indicated by a shift to the left of the vertical borderline between the area of active and inactive firms. The path AB in Fig. 4 depicts the corresponding aggregate macro reaction.

Fig. 3: Application of the modified Mayergoyz/Preisach procedure
– active firms under a volatile interest rate

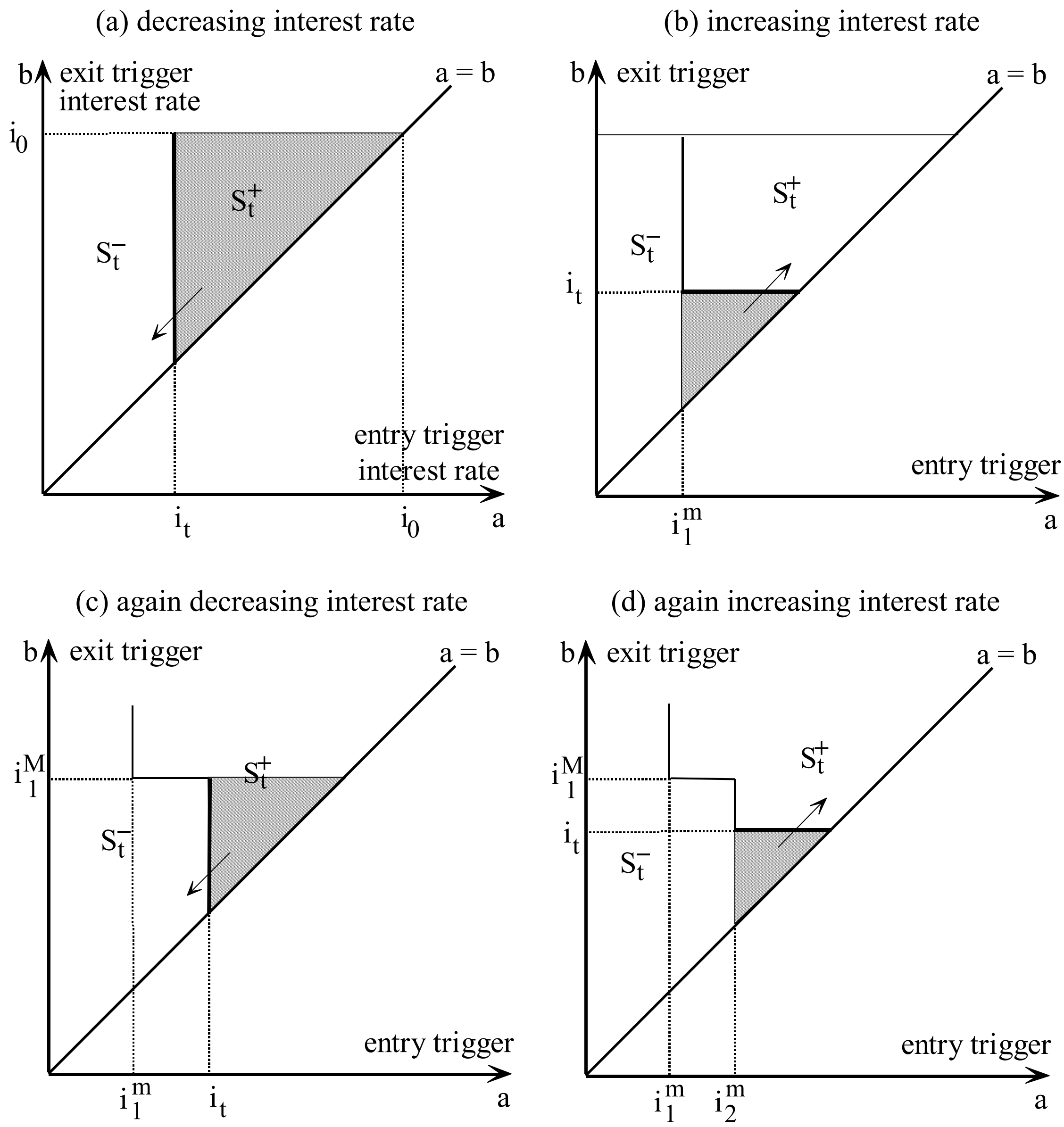
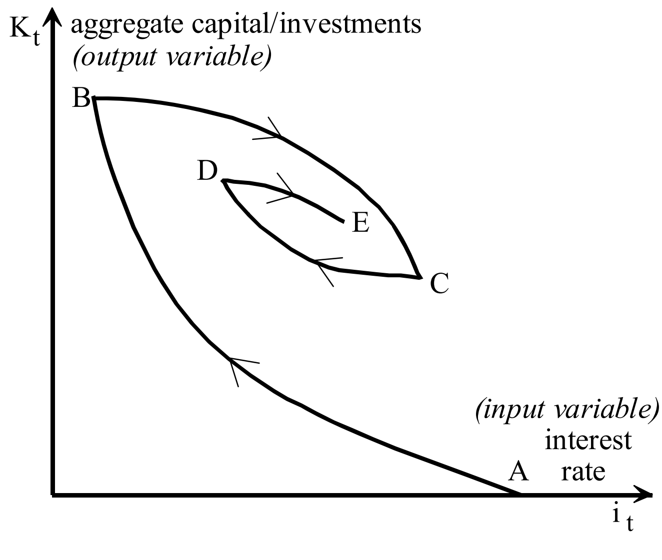


Fig. 3 (b) traces a subsequent increase of the interest rate (i.e. a contractionary monetary policy): i_t rises from the lowest value, the (local) minimum i_1^m. Therefore, the area S_t^+, representing active firms, now shrinks, since firms that have recently invested, now disinvest as the interest rates rises above their exit trigger b_j. For an increasing interest rate, the activity changes (hatched area) are illustrated by a horizontal shift in the lower part of the S_t^--S_t^+-borderline. In Fig. 4 the corresponding path is BC.

If the interest rate falls again after reaching the local maximum i_1^M, area S_t^+ again expands, depicted in Fig. 3 (c) by a vertical shift to the left of the lower part of the borderline. The corresponding macroeconomic reaction is path CD in Fig. 4. The result of the subsequent shifts is a "staircase-shape" of the border between the two areas. If the recently reached (local) minimum is not as low as the lowest minimum i_1^m, a staircase step in the borderline remains – characterised by the coordinates ($a=i_1^m$/ $b=i_1^M$). If the signal level had continued to decrease and had passed the original minimum, the a-coordinate of the "i_1^m-step" would have been *"wiped out"* and replaced (Mayergoyz, 1986, p. 605). However, if the new local minimum is higher than the "old" i_1^m (as traced in Fig. 3 (c)), this "old" minimum remains and the new local minimum becomes the second lowest, labelled i_2^m.

Fig. 3 (d) illustrates a subsequent increase in the interest rate (contractionary monetary policy). The borderline is changed by an upward shift of the lower horizontal part (path DE in Fig. 4). If i_t does not rise above i_1^M a new local maximum would become the second highest maximum i_2^M. If the input were to rise above the "old" i_1^M, the b-coordinate of the corresponding staircase-step would be eliminated. If subsequent local minima and maxima are not as "extreme" as the preceding extrema, a new corner in the staircase border is created. However, local minima which are lower than preceding minima will erase the a-coordinate of the corresponding corners; subsequent local maxima will '*wipe-out'* the b-coordinate of corners corresponding to lower preceding maxima (Amable et al., 1991, pp. 11 ff.).

Fig. 4: The continuous "macroeconomic" hysteresis loop for the aggregate investment activity related to the interest rate



The aggregate system for the entire economy displays a memory of non-erased (*'non-dominated'*) past input (i.e. interest rate) extrema – represented by the "staircases" in the borderline of the area S_t^+ of active firms. Aggregation leads to a change in the pattern of hysteresis. A passing of triggers is necessary at the micro/firm level, in order to induce a permanent effect, whereas *every local extremum* in the time-path of the input variable will have a persisting effect (called "remanence") in the aggregate loop (see Fig. 4). For this reason, this macro hysteresis pattern has been called '*strong'* hysteresis (e.g. Amable et al., 1991).

The *distribution* of the heterogeneous firms in the ($a_j \geq b_j$)-triangle is of course relevant (for the transmission of monetary policy/interest rate changes to the real economy). A *continuous* (called 'strong') macro loop as in Fig. 4 requires a continuous distribution of the firms in the ($a_j \geq b_j$)-region. The exact density and the (a_j,b_j)-distribution of firms determines the curvature of branches of the macro loop. The less heterogeneous the firms are, the more these firms are clustered in a specific area in the (a_j,b_j)-diagram, and as a consequence the more "curved" are the macro branches. In the special case of a multiplicity of homogenous firms represented by the similar point, the macro loop degenerates to a non-ideal relay.

The sunk investment costs of a firm H_j and the uncertainty effects determine the difference between the entry and the exit trigger. The higher the level of sunk costs and of different types of uncertainty, the bigger is the difference between the entry and the exit trigger, i.e. the wider is the "band-of-inaction". Thus firms with high sunk costs and/or affected by a high level of uncertainty (i.e. high option values) will be located far above the 45°-(a=b)-line. The more firms are located far away from the "non-hysteretic" (a=b)-line, the more "inflated" is the macro-loop (i.e. the grater is the distance between an increasing upward branch in a situation of a decreasing interest rate compared to an downward branch in a situation with a rising interest rate. In the special/border case of no sunk costs and no uncertainty, all firm are located on the (a=b)-line, resulting in a standard-type macroeconomic investment function with a negative impact of the interest rate on investments, without a differentiation of "upward" and "downward" directions.

4. Conclusions

The purpose of this paper is to show how microeconomic and, derived from that, also macroeconomic investment depends on (present and future) changes in the interest rate. Since we accept that interest rate changes are driven, at least partly, by central banks, important implications for monetary policy and the monetary policy transmission to the real sector of the macroeconomy (investment) emerge. If interest rate expectations are assumed to be fixed, an unexpected change in the current and future rate of return on capital was shown to result in an investment/disinvestment pattern of a firm which can be described by a so called 'non-ideal relay'. However, if alternatively the interest rate is assumed to be the single input variable, the entry and exit conditions could be solved for the interest rate.

In a next step we dealt with the effects of uncertainty. We derived that a non-ideal relay emerges in a scenario with stochastic revenue changes. Then we demonstrated that a qualitatively similar widening effect on the 'band of inaction' results if uncertainty is not based on stochastic revenue changes, but on stochastic future changes of the interest rate. However, these results necessarily rely on a kind of "mirrored" non-ideal relay pattern. In a stochastic environment, this implies that monetary policy based on *low current interest rates* as currently in the Euro area may *not* be overly *effective in providing an incentive to invest*. Only if the central bank implements monetary policy strategies such as "forward guidance" and is able to credibly commit to low interest rates also for the foreseeable future, our quite strong verdict may be of less relevance (Bernanke/Reinhart, 2004, Eggertsson/Woodford, 2003).

In this regard, we see the Euro area long-term interest rate as the result of the expected patterns for short-term rates which, in turn, are driven by the European Central Bank's inflation forecasts.[4] Reflecting the increasingly active central bank transparency debate in the literature, the ECB has moved in line with other central banks (Kedan/Stuart, 2014) and delivers an *outlook on its future path of policy rates* ("forward guidance", Belke, 2018a). Our model could thus, for instance, contribute to establishing the optimal extent of a central bank's "forward guidance".

However, with respect to forward guidance it can be argued that the ECB does not describe anything else than a *policy rule for its future interest rate path*. Forward guidance then intends to strengthen the credibility of the ECB's monetary policy strategy. Another important caveat is that "forward guidance" cannot correspond with any long-run commitment to a specific interest rate level because this would insinuate an intended change in the medium-term oriented "monetary policy strategy" (Belke, 2018a). If this is true, however, it is even clearly *not appropriate* to speak of a *change in the reaction function* of the ECB driven by "forward guidance". Moreover, the ECB will not be capable to decouple its monetary policies from

[4] Belke (2018a), for instance, argues that the current low-interest rate policy in the Euro area affects also the long-term yields of euro area savings negatively (in contrast to Mario Draghi's view), exactly because long-run interest rates are driven by the sequence of the inflation forecasts by the ECB staff.

those of the Fed during and after the process of “normalisation”. As soon as the crisis is over and the world economy will have entered more “normal times”, the ECB should and most probably will not abide by this instrument. This is so because inflation “forecasts” underlying forward guidance are vexed by much uncertainty and entail risks also for the reputation of central banks (which have to “invest in their reputation under uncertainty”).

What is more, the ECB’s announcements on their “forward guidance” may not necessarily be conceived as credible – for instance due to election dates located between the announcement and the dates for which the underlying inflation outlook is published. In that case, deviations of the markets’ action and the central bank’s ideal projection of market behaviour cannot be excluded. By indicating the need to curb official rates also for the next years could convey the impression that the bank anticipates the crisis lasting for several years to come. But if markets get more pessimistic, consumer and investment spending suffer (Belke, 2018). Seen on the whole, thus, the instrument of “*forward guidance” does not make our considerations regarding the “option value of waiting with investment under uncertainty” less relevant.*

As a final step, we aggregated the micro hysteresis effects to the macroeconomic level. The standard procedure is based on the microeconomic relay-type for revenue variations, where the entry price/revenue trigger has a higher level compared to the exit trigger for all heterogeneous firms. Since this relation is “mirrored” with the triggers being stated in terms of interest rates, we modified the standard Preisach aggregation procedure, with investment/entry triggers of the interest rate below the exit/disinvestment triggers. We then derived the continuous “macroeconomic” hysteresis loop for the aggregate investment activity related to the interest rate.

In this paper, we developed an innovative tool to model a leverage point for monetary policy to impact on macroeconomic investment under uncertainty. This allowed us to draw some conclusions about the effects of low or even zero interest rate monetary policies on macroeconomic investment. Furthermore, this tool may now as a next step be integrated into a fully specified general equilibrium model. However, the main challenge will be to model the feedbacks to the hysteretic sub-system derived here.

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